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ABSTRACT

This compilation consists of a collection of reports on models and studies in the area of manpower management and planning. Part One contains a doctoral dissertation, "The Application of Computer-Assisted Multi-Level Manpower Planning Models in the Federal Government" by R. J. Niehaus, which provides a comprehensive summary of the U.S. Navy's Office of Civilian Manpower Management modeling research program. Parts Two and Three contain papers by R. J. Nichaus, S. Charnes, W. W. Cooper, and others discussing the mathematics of models for aggregate manpower planning and models of the assignment and spectral analyses type. Topics covered by the papers are: (1) "A Goal Programming Model for Manpower Planning," (2) "A Model for Civilian Manpower Management in the U.S. Navy," (3) "A Generalized Network Model for Training and Recruiting in Manpower Planning," (4) "Multi-Level Models for Career Management and Resource Planning," (5) "Static and Dynamic Assignment Models with Multiple Objectives and Some Remarks on Organization Design," and (6) "An Algorithm for Multi-Attribute Assignment Models and Spectral Analyses for Dynamic Organization Design." (SB)

STUDIES IN MANPOWER PLANNING

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PREFACE

This book consists of a collection of reports on models and studies in the area of manpower management and planning. This work was accomplished as part of a research program of the U. S. Navy's Office of Civilian Manpower Management (OCMM). The first part of the book consists of the doctoral dissertation of R. J. Niehaus for the School of Government and Business of the George Washington University. One of the contributions of this dissertation in that it provides a comprehensive summary to date of the OCMM modelling research program. The second and third parts consists of a series of papers by A. Charnes, W. W. Cooper, R. J. Niehaus, and others discussing the mathematics of the models. The second part deals with models for aggregate manpower planning and the third with models of the assignment and spectral analyses type.

There are two others who have contributed to portions of the research which is reported. These are D. Sholtz of OCMM and A. Stedry of The University of Texas. Acknowledgement is also due to the officials of OCMM who supplied guidance and direction for these studies. Among others, these included: R. H. Willey, CAPT. W. Gundlach, J. Cardillo, and P. Meyerson.

In addition to OCMM, other organizations have contributed to the research. They include: The Operations Research Branch and the Personnel and Training Branch of the Office of Naval Research; the Resource Analysis Division, Center for Naval Analyses; the Technical Advisor for Personnel Logistics, Office of Research, Development, Test, and Evaluation, Chief of Naval Operations; and the Special Assistant for Research to the Assistant Secretary of the Navy (Manpower and Reserve Affairs).

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PART I

COMPREHENSIVE SUMMARY

The Application of Computer-Assisted Multi-Level Manpower Planning Models in the Federal Government

by

Richard J. Niehaus

July 1972

Abstract

This study examines the application of large-scale multi-level models for manpower planning in the Federal Government. This includes the data systems which are necessary for their implementation. A multi-level model can be defined as one in w:·'ch more than one level of decision making is included in the same model structure. This study concentrated on the models to link top level resource planning with the career planning process.

This study explored for the first time the applications software necessary to support manpower planning models of the goal programming variety. It includes a number of d.ta studies documenting the initial operational experience of the generalized network form of these goal programming models. New mathematics were developed including a multi-level model which integrates resource goals with manpower goals. This is done by means of examples showing the methodology of transferring manpower model mathematics into workable data systems. It is done in terms of both the technical problems and the managerial problems of such models to support top level policy testing.

Acknowledgement for assistance in the research is due to M. Wofsey and G. Black, School of Government and Business, The George Washington University; A. Charnes, Center for Cybernetic Studies, University of Texas, W. W. Cooper, School of Urban and Public Affairs, Carnegie-Mellon University; and J. Merck, Office of Management and Budget, Executive Office of the President.

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CHAPTER I
ISSUES AND TRENDS

The area of manpower planning has attracted much attention in both the research and managerial literature in the past several years. Interest is shifting away from the use of manpower planning techniques mainly for trend analyses. Focus is upon the use of the manpower planning techniques to support decision and policy making. This shifting emphasis is partly a result of new management science techniques, of large-scale computerized personnel data banks, and of highly efficient computation equipment. More importantly, however, the emphasis comes from the fact that policy and decision makers are becoming more demanding of planning information because of a stronger reliance on a systems approach to problem solving. One new technique which is being added to this growing collection is the multi-level model. This model can be defined as one in which more than one level of decision making is included in the same model structure. For example, one might include top level manpower and financial budgeting in the same model to compute career planning requirements. In this way the program planning aspects of the management process can be made to guide the career planners in a natural and direct manner. Multi-level models could also be constructed to link overall career planning with a system to assist in the operational task of assigning individuals to jobs.

There are reasons why the computer-assisted techniques are not being used to their full potential in the manpower planning area. Among them are the problems of understanding the techniques themselves and of confidence in the data. Another is that the models are new and thus must be shown to be better than current practice. Further, the sheer size of the models (in terms of the number of equations and variables) requires considerable support in terms of input data preparation and analyses. For example, some test examples already contain 3,200 equations with 5,200 variables. Finally, the models can be used to include a very wide spectrum of management activities. For example, the latest versions can be used to integrate career management, training, recruitment planning, and resource allocations into one planning system.

The research involved a number of data studies to document the ways in which the multi-level models can be used to provide useful management information. This part of the research was designed to illustrate the ways in which one might build flexibility and variety into a large-scale data system and still provide the needed control over the system outputs.

The significance of the research is that it provides a methodology that organizations can use to develop flexible manpower planning data systems. It also shows how these data systems can be used once they are built. These are important issues, since the use of personnel data systems beyond routine housekeeping applications is being deferred by many Federal agencies because of the complexity of the problem. What this research shows is how one can make use of current historical data files of personnel data systems to estimate future trends. It also shows how such trend analyses can then be coupled with other data systems to obtain a dynamic projection of manpower requirements with respect to those already on board. It includes both the technical problems and the managerial problems of such models to support top-level policy testing.

The idea of using mathematical and statistical techniques to obtain better information on the manpower requirements has its roots in personnel research started by the Armed Services during World War II.[1] In the early days, these efforts were limited for the most part to the use of classical statistical techniques such as

[1]See *Personnel Research and Systems Advancement*, Proceedings of the Twenty-fifth Anniversary Symposium of the Personnel Research Laboratory, U. S. Air Force, San Antonio, Texas, Nov. 1-3, 1966, PRL-TR-67-33, Dec. 1967.

regression analyses and factor analyses. In more recent years this analytical work has been aided by many orders of magnitude by the use of computers. This use of computers has also exposed many of the technical problems associated with applying classical statistical techniques. This exposure led to the development of models which broadly can be classified into Monte Carlo simulation models and into analytical models.

In the past few years there have been several reviews of these models. Chief among them have been the papers by Huber and Falkner,[1] Walker,[2] Heneman and Setzer,[3] Smith and Lawrence[4] and Charnes, Cooper and Niehaus.[5] A reasonably comprehensive bibliography has been put together by Lewis.[6] Additionally, there have been several international meetings on these models sponsored by the NATO Scientific Committee[7] and by the Institute of Management Sciences.[8] Thus, the literature of manpower planning in an organizational context is beginning to have a firm foundat..n.

The Monte Carlo models can be described as suggested by Hillier and Lieberman[9] as performing sampling experiments on a model of the system. The resulting system is of a sequential interrelated nature lacking in some instances the precision desired. However, some examples of applications of simulation models to manpower planning can be cited. For example, there are the Navy ADSTAP models for enlisted personnel developed by Silverman.[10] There are the so-called entity simulation models of Groover[11] for force planning at the Department of Defense level. Entity simulation models have also been applied by Bottenberg[12] for enlisted personnel planning of the Air Force. Other cases in point, at the micro level of manpower planning where testing of behavioral science propositions is important, are the work of Bonini[13] and that of Weber.[14] Embedding these one-variable-at-a-time derived propositions within a total systems concept has, at least on occasion, produced surprising and even disconcerting results.[15] These developments lead away from the focus of this report. It is concerned primarily with analytical manpower models for aggregate skill and force level planning.

Analytical models differ from Monte Carlo simulation models in that they attempt to abstract the essence of a problem to reveal the underlying structure. In many instances this results in a simultaneous treatment of all facets of the problem. Also, if mathematical programming techniques are employed, the optimizing function is included in the models. As far as manpower planning at the aggregate levels is concerned, analytical models have been developed which can be classified as (1) Markov process models, (2) mathematical programming

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[1]G. P. Huber and C. H. Falkner, "Computer-Based Man-Job Matching: Current Practice and Applicable Research," *Socio-Economic Planning Science*, III (1969), 385-409.
[2]J. W. Walker, "Forecasting Manpower Needs," *Harvard Business Review*, March-April, 1969.
[3]H. G. Heneman, Jr. and G. Setzer, *Employer Manpower Planning and Forecasting*, Manpower Research Monograph No. 19 (Washington: U. S. Department of Labor/Manpower Administration, 1970).
[4]A. R. Smith and J. Lawrence, "Manpower and Personnel Models in the United Kingdom," Presented at 41st National Meeting of the Operations Research Society of America, New Orleans, Louisiana, April 27, 1972.
[5]A. Charnes, W. W. Cooper, and R. J. Niehaus, "Mathematical Models for Manpower and Personnel Planning," *Proceedings of U. S. Naval Personnel Research and Development Symposium on Computer Simulation as Related to Manpower and Personnel Planning*, ed. by A. I. Siegel (Annapolis: April 27-29, 1971).
[6]C. G. Lewis, ed., *Manpower Planning: A Bibliography* (New York: Elsevier Publishing Co., Inc., 1969).
[7]See A. R. Smith, ed., *Models of Manpower Systems* (London: The English Universities Press, 1970).
[8]See D. J. Bartholomew and A. R. Smith, eds., *Manpower and Management Science* (London: The English Universities Press, 1971).
[9]F. S. Hillier and G. L. Lieberman, *Introduction to Operations Research* (San Francisco: Holden-Day Inc., 1967).
[10]J. Silverman, "Personnel Resource Planning in an Operational Environment," presented at NATO Conference, "Manpower Planning Models," Cambridge, England, Sept. 6-10, 1971.
[11]R. O. Groover, "PERSYM: A Generalized Entity-Simulation Model of a Military Personnel System," presented at NATO Conference, "Mathematical Models for the Management of Manpower Systems," Porto, Portugal, Sept. 1-5, 1969.
[12]R. Bottenberg, "Models for the Simulation of the Distribution of Military Personnel," Presented at the 41st National Meeting of the Operations Research Society of America, New Orleans, La., April 27, 1972.
[13]C. P. Bonini, *Simulation of Information and Decision Systems in the Firm* (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1964).
[14]W. Weber, "Manpower Planning in Hierarical Organizations: A Computer Simulation Approach," *Management Science*, Nov. 1971.
[15]J. Forrester refers to such results as "counter-intuitive" in *Urban Dynamics* (Cambridge, Mass.: Massachusetts Institute of Technology Press, 1969).

models, (3) activity analyses and input-output types of models, and (4) various combinations of these approaches. The multi-level are in the last category, where several of these techniques are brought together into one modeling system.

Markov Models

There have been a variety of recent applications of stochastic models of the so-called Markov matrix type to manpower planning. These Markov models generally multiply a vector of personnel in various job categories by a matrix of transition rates. This allows one to obtain a projection of the current workforce based upon past trends. Probably the most well known applications are those of Bartholomew,[1] of Vroom and Macrimmon,[2] and of Merck.[3] Among the others one could cite on the use of Markov models for manpower planning are Forbes,[4] Butler,[5] Rowland and Sovereign,[6] and Marshall and Oliver.[7] A form of the Markov models was also suggested by Kane[8] to check out the "Peter Principle." All of this suggests that Markov models contain an essential element for developing manpower projections. This turns out to be that the transition matrix allows one to interconnect the internal and external manpower flows across time periods. This leads to dynamic models of the Markov decision variety.

The method of embedding Markov models into a mathematical programming decision model (i.e., one that optimizes some set of decision criteria) was first reported in 1967 by Charnes, Cooper, and Niehaus.[9] Before looking in somewhat more detail at these Markov decision models, some of the other applications of mathematical programming to manpower planning will be examined.

Mathematical Programming Models

The first applications of linear programming to manpower planning (other than static assignment models) did not produce satisfactory results. Since they were generally cost minimizing models, they recommended hiring all low-cost personnel (such as janitors or privates) and firing all high-cost personnel (such as executives and generals). An early contribution is the work of Charnes, Cooper, and Ferguson.[10] In a model they designed for the General Electric Company to assist in setting executive compensation, they developed the concept called "goal programming." Here, the idea is to try to hit a number of management targets "as closely as possible," subject to a set of underlying constraints. The management targets in this case were the salary levels of the executives. The constraints were the attributes possessed by the individuals involved.

The goal programming models are essentially an extension of the ideas of regression analyses. The models are non-linear in formulation, but are transformed to a linear equivalent for optimization. This feature is quite important, since access is immediately secured to existing linear programming computer codes. A critical part of the problem of implementation can then be overcome as far as computer software is concerned.

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[1]D. J. Bartholomew, *Stochastic Models for Social Processes* (London: John Wiley & Sons, 1967).

[2]V. H. Vroom and K. R. MacCrimmon, "Toward a Stochastic Model of Managerial Careers," *Administrative Sciences Quarterly*, March, 1968.

[3]J. A. Merck, "A Markovian Model for Projecting Movements of Personnel Through a System," Defense Documentation Center, AD 616704, March, 1965.

[4]A. F. Forbes, "Markov Chain Models for Manpower Systems: An Example and Some Comments in Testing for Goodness of Fit," presented at 17th International Meeting of the Institute of Management Sciences, London, July 1-3, 1970.

[5]A. D. Butler, "A System for Equalizing the Promotion Rates between Grades within Similar Hierarchies," presented at NATO Conference, "Manpower Planning Models," Cambridge, England, Sept. 6-10, 1970.

[6]L. Rowland and M. Sovereign, "Markov Chain Analysis of Internal Manpower Supply," *Industrial Relations*, IX, No. 1, October 1969.

[7]R. T. Marshall and R. M. Oliver, "A Constant Work Model for Student Attendance and Enrollment," Office of the Vice President–Planning and Analysis, University of California Research Report No. 69-1, February, 1969.

[8]J. Kane, "Dynamics of the Peter Principle," *Management Science*, August, 1970.

[9]A. Charnes, W. W. Cooper, and R. J. Niehaus, "A Goal Programming Model for Manpower Planning," in *Management Science in Planning and Control*, ed. by John Blood (New York: Technical Association of the Pulp and Paper Industry, 1968).

[10]A. Charnes, W. W. Cooper, and R. Ferguson, "Optimal Estimation of Executive Compensation by Linear Programming," *Management Science*, 1. No. 2, January 1955, 423-430.

The applications software development then becomes one of providing the appropriate input data and of translating the linear programming outputs into management reports.

In addition to the goal programming models, there have been other manpower models which suggest the use of mathematical programming. Early theoretical work was accomplished by Kossack and Beckwith[1] for the U. S. Air Force. More recently, a linear programming model was included in a mode by Kildebeck, Kipnis, and Mackey,[2] aimed at the pilot training cycle of the Air Force. The Marine Corps as described by Marsh[3] is using a linear programming model to assist in the planning of troop rotations. Similar models were suggested by Morgan[4] for use by the Royal Air Force, and by Clough, Dudding, and Price[5] for the Canadian Forces. Industrial manpower models utilizing mathematical programming include the work of Purkiss[6] for the British Iron and Steel Institute and that of El Agizy[7] for IBM. Most of these models have experienced implementation difficulties. In addition to the problem of management communication, their implementation has been slowed by the model constructions themselves. Generally, they have not handled the problem of multiple period planning very well. In some, the resolution of this problem is attempted by using multiple objective functions. In others resolution is made by making the transition rates the decision variables of the resultant linear program. What they all indicate, however, is that the manpower planning problem, almost of necessity, is one which involves many variables and equations.

The Markov decision models developed by Charnes, Cooper, and Niehaus[8] combine the strengths of the Markov models with that of the goal programming models. The goal programming mathematics with embedded Markov processes allow a dynamic evaluation of manpower requirements in relation to the manpower inventory. It makes possible the ability to plan probabilistically for a sequence of planning intervals. It also enables one to deal with the various goals of manpower planning in the natural setting of existing policies and practices. This can be done without insisting upon any preordained consistency between the various manpower goals. Finally, it makes it possible to extend the models to accommodate budgetary (financial) planning with manpower planning, as twin aspects of a simultaneous decision process.

Activity Analyses and Input-Output Models

Another type of analytical model which has been employed in conjunction with manpower planning is the input-output and activity analyses type. The activity analysis model differs from the input-output model in that it allows for a choice between alternatives. Also, activity analysis models allow for categorizations that do not fit easily into the standard input-output format. On the other hand, these activity analysis models require categories and constructions that can sometimes make the resulting models unwieldly and difficult to use in particular circumstances. Another difficulty which has been pointed out by Miernyk[9] and others in that these models do not adequately deal with the dynamic interrelationships involved in multi-period planning. The point to bear in mind is that both input-output and activity analysis models were originally designed by

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[1] C. F. Kossack and R. E. Beckwith, "The Mathematics of Personnel Utilization Models," Purdue University Contract, AF 41 (657), 160WADC-TR-359, Arlington, Virginia, ASTIA, November 1959.

[2] J. Kildebeck, G. M. Kipnis, and W. E. Mackey, "Rated Resources Requirements System," Headquarters, U. S. Air Force, Washington, August 15, 1969.

[3] J. W. Marsh, "Information Requirement for Managing the Manpower of a Large Organization," Presented at 41st National Meeting of the Operations Research Society of America, New Orleans, La., April 17, 1972.

[4] R. W. Morgan, "Manpower Planning in the Royal Air Force: An Exercise in Linear Programming," presented at NATO Conference, "Mathematical Models for the Management of Manpower Systems," Porto, Portugal, Sept. 6-10, 1969.

[5] D. J. Clough, R. C. Dudding, and W. L. Price, "Mathematical Programming Models of a Quasi-independent Subsystem of the Canadian Forces Manpower System," in *Models of Manpower Systems*, ed. by A. R. Smith (London: The English Universities Press, 1970).

[6] C. J. Purkiss, "Approaches to Recruitment, Training, and Redeployment Planning," presented at NATO Conference, "Manpower Research in the Defense Context," London, August 14-18, 1967.

[7] M. El Agizy, "A Stochastic Programming Model for Manpower Planning," IBM Corporation, Armonk, N. Y., presented at 17th International Meeting of the Institute of Management Sciences, London, England, July 1-3, 1970.

[8] A. Charnes, W. W. Cooper, and R. J. Niehaus, "A Goal Programming Model for Manpower Planning," in *Management Science in Planning and Control*, ed. by John Blood (New York: Technical Association of the Pulp and Paper Industry, 1968).

[9] W. H. Miernyk, *The Elements of Input-Output Analysis* (New York: Random House, 1969).

Leontief[1] and Koopmans[2] to conform closely with the rubrics and requirements of general equilibrium analysis in formal economics. These models are also related to the process analysis models discussed by Manne and Markowitz.[3] Manpower models utilizing process analysis have been suggested by Blanding, DeHayes, and Taylor.[4] However, these model constructions appear to have had implementation problems due to some of the difficulties discussed above. This is not to say that these modeling efforts are unimportant. With modification, there is strong likelihood that they could be quite useful in the area of manpower and personnel planning.

An example of an input-output model which is being successfully implemented is the Navy Requirements Model (NARM). This model is being developed by Augusta[5] of the Center for Naval Analyses for the General Planning and Programming Division of the Chief of Naval Operations. The NARM model is being used for study of cost allocations in the Navy. It is worth noting that this model is being used to assist in the preparations of a significant portion of the Fiscal Year 1974 budget. A limitation of this model is the fact that it provides static rather than dynamic projections. It is an aggregate planning tool and does not directly tie back to the skills planning process.

Input-output models for manpower planning are part of the work of Kovacs[6] in Hungary. This model is being used to assist in the planning of enrollment in the universities in Hungary.

Multi-Level Models

Turning to the possible extension of these input-output approaches to dynamic models with additional constraints and alternatives, as discussed by Charnes, Cooper, Niehaus, and Sholtz,[7] the possibility of a true multi-level model is realized. Their work includes the joining of an input-output model for resource planning with a career management model for manpower planning. Goal programming features are included to make it possible to handle inconsistencies in resource availabilities and other requirements. This model has been arranged so that resources allocation is accorded greater weight in the objective than career management. The result is a model which is multi-level in the sense that two different levels of decision making are considered simultaneously in the same model.

Another multi-level model for manpower planning is the one proposed by Kildebeck, Kipnis, and Mackey.[8] This model is actually a multi-stage model in that the results of the computations at the top level of decision making are used to constrain the subsequent computations at the next more detailed level of decision making. Here, a Monte Carlo simulation model is suggested for the resource allocation decisions followed by a linear programming model for the manpower planning decisions. This modeling system has not been implemented beyond initial prototype computations and to the knowledge of the researcher is not being pursued further.

Other multi-stage and multi-model systems have been proposed to assist in the manpower planning process. Among them is the system suggested by Williams and Ozkaptan.[9] These ideas are still in the very early conceptual stage without explicit model structures or numerical prototypes to demonstrate fully their feasibility. All of this and the preceding discussion indicate that to the researcher's knowledge there are no working

[1]W. W. Leontief, *The Structure of the American Economy 1919-1939* (New York: Oxford University Press, 1951).
[2]T. C. Koopmans, ed., *Activity Analysis of Production and Allocation* (New York: John Wiley & Sons, Inc., 1951).
[3]A. S. Manne and H. S. Markowitz, eds., *Studies in Process Analysis* (New York: John Wiley & Sons, Inc., 1963).
[4]S. W. Blanding, D. W. DeHayes, Jr., and J. G. Taylor, "The Use of Process Analysis to Forecast Manpower Requirements," presented at Operations Research Society of America Meeting, Miami, November, 1969.
[5]J. H. Augusta, R. A. Jenner, and G. W. Ryhanych, "Interim Input-Output Resource Allocation Model," Center for Naval Analyses Research Contribution, No. 134, March 2, 1970.
[6]J. Kovacs, "A Model for Planning School Enrollment," Budapest: Institute of Economics, Hungarian Academy of Sciences, 1969).
[7]Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models."
[8]S. Kildebeck, G. M. Kipnis, and W. E. Mackey, "Rated Resources Requirements System" (Washington: Headquarters, U. S. Air Force, August 15, 1969).
[9]H. Williams, and H. Ozkaptan, "U. S. Navy Shore Manpower Planning System," presented at NATO Conference, "Manpower Planning Models," Cambridge, England, September 6-10, 1971.

examples of analytic multi-level manpower planning models beyond that reported by Charnes, Cooper, Niehaus, and Sholtz.[1]

Most of the manpower modeling in the Federal Government has been conducted by the Department of Defense. With the exception of the work sponsored by the Navy's Office of Civilian Manpower Management, little attention has been paid to modeling Civil Service manpower. This point has been substantiated by Clark[2] of the U. S. Civil Service Commission. A similar conclusion was reached by the U. S. Government Inter-Agency Task Force on Personnel and Computers in Decision-making.[3] This is to be contrasted to the British Civil Service Department which, as reported by Smith,[4] has had a strong manpower modeling research effort for several years.

Model Implementation

Paralleling or following the development of the model structures have been the efforts to develop the data systems to support the models. It is how this effort is organized and developed that spells the ultimate success or failure of a particular modeling system. Highly sophisticated systems involving large amounts of financial support have failed due to faulty management of the implementation process. Some were replaced with less complex systems, since the organization was not ready or capable of using the new capability. Others failed because the developers attempted to accelerate the implementation before the technology was adequately tested. The point here is that most manpower modeling systems have a tricky initial inertial problem to overcome.

With many manpower systems, the initial inertial problem is due in a large part to the model structures themselves. The models may contain the basic technological improvement to warrant further implementation. However, most likely there are many minor structural deficiencies, which need correction. It is these minor problems, which tend to generate the need for considerable unplanned computer software, that stifle the overall development process. It is during this part of the process that the systems also need to be generalized to handle the anticipated variety of data demands. All of this contributes to the large amounts of computer software needed to move the models to an operational environment.

The most critical period in the modeling system's life is the time between successful demonstration of the small prototypes and the completion of the first version of the operational system. Interest has already reached its first peak, and the system may not be able to deliver for some time to come. At this point the system may lose some of its chief followers and patrons. Part or all of the implementation resources may also be withdrawn due to the criticisms which are likely to appear. This may result in one or more of the key people leaving or being transferred to other projects. The system then languishes and dies as new technology replaces the edge the system once had. This points to the need for the long term continuation of the team of key people, if the system is to survive. It also emphasizes the need for continually introducing new technology to the system as it is being implemented.

There are many tactical considerations which need to be taken into account during the implementation process. First, there is the need for the development of a few useful outputs very early in the system's life. These should be the outputs of one or more of the critical sub-systems. In this way a two-way dialogue can be established between the system's developers and the system's users. Then, as the more complicated outputs become available, there will be a much quicker acceptance of their potentials. It also gives the non-technical evaluators some background with which to judge the system. Another tactical consideration is the development of prototypes. There is a very good likelihood that the results will not be accepted by the organization.

[1]A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "Multi-Level Models."

[2]H. L. Clark, "Problems and Progress in Civil Service Manpower Planning in the United States," presented at NATO Conference, "Manpower Planning Models," Cambridge, England, September 6-10, 1971.

[3]C. Smith, *Report of Task Force on Personnel and Computers in Decision-making*, IAG 204 (Washington: U. S. Civil Service Commission, December 1970).

[4]A. R. Smith, "Some Problems of Manpower Systems Analysis in the U. K. Civil Service," presented at NATO Con ence, "Manpower Planning Models," Cambridge, England, September 6-10, 1971.

However, they provide the information for needed improvements. They also switch the critics from arguing techniques to arguing data. Assuming the techniques will provide a superior improvement, good administrative practices can be implemented to improve the data quality. The important point here is that the prototypes allow the investment to be kept within reason while all of these implementation problems are being solved.

CHAPTER II

DESCRIPTION OF A MULTI-LEVEL MODEL

The purpose of this chapter is to provide a concise description of a multi-level model. This will be done by discussing first the career management portion of the model. Included will be a numerical example using Navy data. This will be followed by a discussion of the complete structure of the multi-level model. This chapter will conclude with a review of the numerical example presented by Charnes, Cooper, Niehaus, and Sholtz[1] at the NATO Conference "Manpower Planning Models."

The design of the multi-level manpower planning model implies an integrated approach to career management and resource planning. A case in point is the OCMM multi-level model. It was designed explicitly to bring career management into contact with the program planning or PPB systems in being or under development within the Navy. The importance of this work can be brought into prominence by observing that resource planning is actually the driving force behind the manpower allocation and career management process.

The OCMM multi-level model consists of a coupling of a generalized network model for manpower planning[2] with an input-output model for resource planning. The generalized network model with appropriate goal equations has been named the OCMM career management model. This model can either stand alone or be linked with an input-output model to form the multi-level model.

Career Management Model

A description of the career management model is given in Figure 1. The objective of the model is to minimize discrepancies from manpower requirements by job category and to favor on-board manpower and new hires over excess personnel or Reduction in Force (RIF's). This objective is accomplished by a scheme of relative weighting factors. These weights are set so that a penalty is paid whenever the manpower requirement (goal for each job category) is not met. A penalty is also paid for hiring or firing. Additionally, the penalty for excess personnel is greater than that for new hires. However, the cost of maintaining the personnel already on-board is set equal to zero These weighting factors allow the model to favor on-board manpower over new hires and (even to a greater extent) over RIF's. Note, that a given set of manpower requirements need only be met "as closely as possible" and that an increasing penalty is paid as one moves away from the goals. The scheme of relative weights can be set so that RIF's have a higher penalty than goal deviations or vice versa. This provides the option to the model user of examining either the policy of "RIF's as a last resort" or the policy of the strict adherence to a given set of manpower requirements. Further, these strategies can be mixed so that some of the manpower requirements will be met much more closely than others. For example, a certain number of executives may be required in all cases.

The objective of the model is subjected to a number of constraints. First, the number on-board in each job category at the start is set equal to a constant. This ensures that the base period population will be completely accounted for in the model solution. The base period population is then submitted to a matrix of movement or transition rates which distinguishes probabilistically between those staying in a particular job category, those being promoted, and those leaving the organization.

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[1]This numerical example can be found in Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models."

[2]For the initial mathematical development of this model see A. Charnes, W. W. Cooper, and R. J. Niehaus, "A Generalized Network Model for Training and Recruiting Decisions in Manpower Planning," in *Manpower and Management Science*, D. Bartholomew and A. R. Smith Eds. (London: The English Universities Press, 1971).

OBJECTIVE: (a) MINIMIZE DISCREPANCIES FROM CIVILIAN MANPOWER REQUIREMENTS BY JOB CATEGORY
(b) FAVOR ON-BOARD MANPOWER AND NEW HIRES OVER REDUCTIONS-IN-FORCE (RIF's)

SUBJECT TO:

NUMBER ON BOARD IN EACH JOB CATEGORY IN EACH PERIOD	- POSSIBLE AMOUNT OVER	+ POSSIBLE AMOUNT UNDER	=	MANPOWER REQUIREMENTS BY JOB CATEGORY
NUMBER ON BOARD IN EACH JOB CATEGORY AT BASE PERIOD			=	INITIAL POPULATION
NUMBER REMAINING IN EACH JOB CATEGORY	+ NUMBER ON BOARD IN EACH JOB CATEGORY AT PRESENT PERIOD	- NEW HIRES + RIF's	=	0
SUM OF (NUMBER ON BOARD IN EACH JOB CATEGORY	X SALARY OF JOB CATEGORY)		≤	TOTAL SALARY BUDGET
SUM OF NUMBER ON BOARD IN EACH JOB CATEGORY			≤	TOTAL MANPOWER AVAILABLE

Figure 1. OCMM CAREER MANAGEMENT MODEL

Another constraint used in the model concerns the total manpower limitation or ceiling. Here, the condition is set that the sum of the number in each job category in a given period must be less than or equal to the total manpower limitation for that period. Budgetary constraints can also explicitly be included. This is done by equations which require that the sum of the number in each job category multiplied by the mean salary of the job category must be less than or equal to the total salary budget.

The career management model can be understood better through a numerical example. The data for this example were obtained from the Five Year Navy Civilian Manpower Plan (FYNCIMP) system.[1] It should be noted that this is the first publication of numerical results using this particular model structure. Earlier published data used models designed prior to the generalized model which was proposed in 1970.[2]

The numerical example to be illustrated involves the major occupation groups (i.e., Professional and Technical, Managerial and Administrative, etc.) of the Navy's white collar work force. These data are further subdivided into grade level groupings (i.e., GS 1-4, 5-8, etc.). Data on the initial population and the five year manpower requirements for this example are given in Table 1. These data should be regarded as experimental.

The transition data shown in Table 2 were developed from the Navy's data files on civilian personnel. They are developed by a series of computer programs for comparing the data files at two points in time. For example, these data show that of the 14,395 personnel in the 30120 job category (i.e., Professional and Technical GS 5-8), 67 percent remained in that category, 16.3 percent transferred to the 30130 category and so on. Research is under way by the Navy to provide means of adjusting these rates to correspond more closely to probable future trends.

Transition data for the June, 1970-June, 1971 period were used in the model to compute the internal movements of on-board personnel over time. This includes for the later periods all of the personnel projected

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[1] A detailed description of the FYNCIMP system will be given in the next chapter.

[2] For a discussion of one of the earlier numerical examples see A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "A Systems Approach to Manpower Management and Planning," *The Journal of Navy Civilian Manpower Management*, Vol. IV, No. 4, Winter, 1970.

TABLE 1

MAJOR OCCUPATION GROUP PROJECTED REQUIREMENTS
GENERAL SCHEDULE PERSONNEL

Major Occupation Group and Grade Level	On-Board 30 June 1971	30 June 1972	30 June 1973	30 June 1974	30 June 1975	30 June 1976
Professional and Technical						
GS 1-4	2,884	2,873	2,913	2,940	2,886	2,859
5-8	14,507	14,450	14,653	14,791	14,514	14,383
9-12	45,995	45,820	46,469	46,904	46,033	45,610
13-15	16,635	16,572	16,803	16,962	16,648	16,496
16-18	304	304	304	304	304	304
Managerial and Administrative						
GS 1-4	63	63	62	62	59	58
5-8	3,768	3,726	3,685	3,653	3,532	3,460
9-12	9,162	9,090	8,989	8,914	8,617	8,443
13-15	2,421	2,395	2,372	2,348	2,270	2,224
16-18	28	28	28	28	28	28
Clerical						
GS 1-4	37,283	36,105	35,958	35,595	34,597	34,079
5-8	21,425	20,749	20,658	20,455	19,881	19,585
9-12	1,865	1,806	1,799	1,781	1,731	1,705
Service						
GS 1-4	6,037	5,846	5,789	5,930	5,732	5,617
5-8	3,271	3,168	3,137	3,214	3,107	3,044
9-12	237	231	229	234	227	222

Experimental Report

as new hires in the earlier periods. The transition data for the years 1972 and 1973 were adjusted to reflect lower exit rates from the Navy due to the lessened job opportunities on the outside. For the years 1974 through 1976, the June, 1970-June, 1971 transition data were used on the assumption that the labor market would again allow more freedom of movement.

The model input data were then arranged into the format of the linear programming matrix shown in Figure 2. No budgetary data were included in this particular example. A solution to the linear programming problem was then obtained. The results are shown in Table 3.[1]

These projections are the first of a kind for this level of aggregation. The data were provided on a tentative basis to the General Plans and Programming Division of The Chief of Naval Operations. They were

[1]It should be noted that the transition data do not reflect the greater number of retirements expected in the 1972-1976 period. However, expected retirement data will be used to modify the transition rates in future computer runs.

RELATIVE FREQUENCY OF MOVEMENT INTER-STATE
GENERAL SCHEDULE PERSONNEL MAJOR OCCUPATION GROUPS
FROM JUN 70 TO JUN 71
TABLE 2

SEX BOTH — BIRTHDATE 000 TO 000 — SERVICE COMPUTATION DATE 000 TO 000 — MAJOR CLAIMANT — ACTIVITY — GRADE LEVEL

OCCUPATION	NO AT JUN70	30120	30130	30140	30150	30220	30230	30240	30250	30310	30320	30330	30340	30350	30410	30420	30430	30440	30450	EXITS
30120	14395	.670	.163			.004	.002			.004	.010									.145
30130	45519	.002	.874	.025			.003					.002								.093
30140	16474		.005	.909	.001			.002					.003							.080
30150	304			.013	.878															.109
30220	3778	.010	.004			.724	.106			.004	.046	.002								.103
30230	9927	.001	.013	.001		.003	.859	.010		.001	.002	.004								.107
30240	1851		.001	.009			.004	.884	.002				.004							.097
30250	13								.846											.154
30310	38671	.009				.002				.694	.066				.001					.228
30320	22226	.022	.003			.011	.002			.012	.816	.007								.128
30330	1933	.006	.061	.001		.002	.022	.001		.001	.006	.804	.006							.091
30340	563		.012	.048				.002				.002	.842							.094
30350	15							.067						.800						.133
30410	6403	.001								.010					.766	.052				.170
30420	3471	.002	.001			.001				.001	.001				.009	.872	.007			.108
30430	253		.008									.004				.012	.870			.107
30440	4			.250													.250	.500		
30450	0																			
I GAINS		462	4	0	0	5	0	0	0	145	51	0	0	0	2	0	0	0	0	
E GAINS		2646	3229	497	13	271	416	51	0	8682	1106	74	15	3	1295	124	17	0	0	
NO AT	JUN71	13735	45789	16658	299	3436	9615	1826	14	36084	22213	1922	571	15	6265	3496	268	3	0	

TOTAL AT JUN 70 IS 165,800 TOTAL AT JUN 71 IS 162,209

	POSITIVE GOAL DISCREP	NEGATIVE GOAL DISCREP	ON-BOARD MANPOWER	NEW HIRE MANPOWER	EXCESS MANPOWER	SIGN	RHS
RELATIVE PRIORITIES	α	α		β	β		
MANPOWER GOALS	-I -I	I I	I I			= =	MANPOWER REQUIREMENTS
MANPOWER ATTRITION			I -M I -M I	-I -I	I I	= = =	INITIAL POP 0 0
TOTAL MANPOWER CONSTRAINTS			1111 1111			$\leq$ $\leq$	TOTAL MANPOWER AVAILABLE
SALARY BUDGET CONSTRAINTS			($/m)1 ($/m)2			$\leq$ $\leq$	TOTAL SALARY BUDGET

Figure 2. CAREER MANAGEMENT MODEL LINEAR PROGRAMMING MATRIX

the best data available for use in a five year forecast of total civilian and military manpower. In the results there have been goal deviations in some of the categories. This is a result of the fact that some of the goals had been set below the employment levels required to maintain the policy of reductions by attrition alone. Some other tentative conclusions are:

1. Within the parameters of this study, it appears that recruitment needs will be greatest in FY 1973 and FY 1974. This will be especially true in the professional and technical occupations.

2. The hiring pattern among grades in the clerical occupations will not change much over the next five years.

3. Year-to-year variations in intake by grade grouping will in many cases be substantial. For example, annual recruiting requirements for GS 9-12 Professional and Technical workers will jump by more than 2,900 from 1972 to 1973 and drop by 1,300 from 1974 to 1975.

4. Changes in total employment have more impact on recruiting requirements in professional-technical and managerial-administrative groups. This might be expected in view of the much lower loss rates in these areas.

Multi-Level Model

The OCMM multi-level model incorporates the career management model as one of its major sub-models. The version of the multi-level model presented at the NATO Conference at Cambridge does not have the goal programming features for the manpower categories. In that version, goal programming was used only in the resource planning or input-output model. The remainder of this chapter will be a review of the example using hypothetical data which was presented at the NATO Conference. In a later chapter, a state-of-the-art extension of this model will be presented. This extension involves the inclusion of goal programming features in *both* the resources planning and career management sub-models.

It was felt by Charnes, Cooper, Niehaus, and Sholtz[1] that the multi-level models should reflect the program planning and budgeting systems of the Navy. The development of PPB systems beyond the methodology

[1]Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 4.

of allocating civilians is not the direct responsibility of OCMM. An investigation was made, therefore, of current and proposed methodologies in the Office of the Secretary of Defense as well as within the Navy. This led to the Navy Requirement Model (NARM) of Augusta *et. al.*[1] of the Center for Naval Analysis. As mentioned in the previous chapter, this model uses the concepts of input-output analyses.

A study was made of the NARM model, including discussions with its designers. These studies indicated that there was a close enough relationship between the CNA and OCMM models to proceed with further developments which offered possible greater liaison between the two approaches. As a start in these developments, it was observed that the Markov matrices embedded in the OCMM model and the resource transfer rates of an input-output model such as the NARM model share the so-called Minkowski-Leontief property in common. Each element of the input-output table is non-negative and each row sums to one in the same manner as a Markov matrix. Also, from a practical viewpoint, these input-output elements can be computed from relatively easily obtained historical data. The goal programming model which Charnes, Cooper, Niehaus and Sholtz presented incorporated these strengths of the NARM model.

In the OCMM multi-level model the input-output transfer rates provide the ability to examine simultaneously relationships between resource producers (e.g., the naval shore establishment) and final resource users (e.g., the fleet). The model also uses the generalized network structure of the OCMM career management model for depicting the dynamic interactions between manpower "inventories" and manpower "requirements." These systems are joined together by a series of "coupling conditions" so that imbalances in one of the systems will be reflected in the other. The goal programming aspects also supply the flexibility for examining the effects on final user demands of changes in resource inputs as well as changes in manpower requirements.[2] The OCMM multi-level model was extended by Charnes, Cooper, Niehaus, and Sholtz[3] to include probabilistic or risk-related considerations. This was done by the use of chance-constrained programming to deal with risk variations in the right hand sides of the resultant linear programs.

Data availability was a major consideration in the construction of the OCMM multi-level model. The model brought together two very large scale systems and the data necessarily had to come from existing or planned automated information systems. These again pointed to the CNA models which use the Navy Cost Information System (NCIS) data tapes reflecting the Navy's portion of the Five Year Defense Program (FYDP). Additionally, the multi-level model was designed to rely upon already developed computer programs to obtain the manpower requirements and transition rates of OCMM's Navy Automated Civilian Manpower Information System (NACMIS).

For computational reasons the multi-level model was also designed to use linear programming for solution purposes. Note that even though this goal programming model is stochastic and non-linear, it is possible to convert it to a linear equivalent for optimization purposes. This feature was preserved even in the extensions utilizing chance-constrained programming to accommodate risk-related considerations. This use of linear programming provides immediate access to the solution routines and sensitivity checks available with the software generally provided with medium and large scale computers.

A description of the model structure as presented at the NATO meeting is given in Figure 3. In the multi-level model the objective is to minimize the weighted deviations from the final user requirements so that in goal programming form these are to be met "as closely as possible." In this case the deviations are accorded relative weights which reflect priorities associated with being over or under each of the final user requirements. These relative weights, which replace the dollar cost normally associated with conventional linear programming models, can be considered a "priority cost" of each of the final user requirements where the highest relative cost is associated with the most critical requirement.

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[1] J. H. Augusta, R. A. Jenner, and G. W. Ryhanych, "Interim Input-Output Resource Allocation Model," Center for Naval Analyses Research Contribution No. 134, 2 March 1970.

[2] It turns out that the classic input-output model which is used in the NARM model can essentially be made a special case of this goal programming model when there is perfect balance between the inputs and the outputs.

[3] Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," pp. 29-34.

TABLE 3

MAJOR OCCUPATION GROUP RECRUITING REQUIREMENT
PROJECTIONS GENERAL SCHEDULE PERSONNEL

Major Occupational Group and Grade Level	On-board June 30, 1971	June 30, 1972 Goal	June 30, 1972 Projected On-board	June 30, 1972 Recruiting Requirement	June 30, 1973 Goal	June 30, 1973 Projected On-board	June 30, 1973 Recruiting Requirement	June 30, 1974 Goal	June 30, 1974 Projected On-board	June 30, 1974 Recruitin Requireme
Professional and Technical										
GS 1-4	2,884	2,873	2,729	1,277	2,913	2,902	1,358	2,940	2,940	1,313
5-8	14,507	14,450	14,450	2,814	14,653	14,653	3,5[illegible]9	14,791	14,791	3,563
9-12	45,995	45,820	45,820	729	46,469	46,469	3,661	46,904	46,904	3,499
13-15	16,635	16,572	17,047	0	16,803	16,803	100	16,962	16,959	464
16-18	304	304	304	10	304	304	20	304	304	20
Managerial and Administrative										
GS 1-4	63	63	63	24	62	62	22	62	62	10
5-8	3,768	3,726	3,726	468	3,685	3,685	591	3,653	3,653	590
9-12	9,192	9,090	9,090	54	8,989	8,989	527	8,914	8,914	542
13-15	2,421	2,395	2,455	0	2,372	2,372	7	2,348	2,348	71
16-18	28	28	28	0	28	28	0	28	28	0
Clerical										
GS 1-4	37,283	36,105	35,436	10,591	35,958	35,958	10.834	35,595	35,595	10,103
5-8	21,425	20,749	20,943	0	20,658	20,658	836	20,455	20,455	828
9-12	1,865	1,806	1,806	0	1,799	1,310	0	1,781	1,781	44
Service										
GS 1-4	6,037	5,846	5,846	1,343	5,789	5,789	1,244	5,930	5,930	1,496
5-8	3,271	3,168	3,246	0	3,137	3,137	0	3,214	3,206	166
9-12	237	231	237	0	229	229	0	234	234	13

TABLE 3

MAJOR OCCUPATION GROUP RECRUITING REQUIREMENTS
PROJECTIONS GENERA[illegible] SCHEDU[illegible]E PERSONNEL

	June 30, 1973			June 30, 1974			June 30, 1975			June 30, 1976		
nt	Goal	Projected On-board	Recruiting Requirement	Goal	Projected On-board	Recruiting Requirement	Goal	Projected On-board	Recruiting Requirement	Goal	Projected On-board	Recruiting Requirement
	2,91[illegible]	2,902	1,358	2,940	2,940	1,313	2,886	2.886	1,246	2,859	2,859	1,252
	14,653	14,653	3,589	14,791	14,791	3,563	14,514	14,[illegible]4	3,195	14,383	14,383	3,284
	46,469	46,469	3,661	46,904	46,904	3,499	46,033	46,033	2.228	45,610	45,610	2,623
	16,803	16,803	100	16,962	16,959	464	16,648	16,648	0	16,496	16,496	155
	304	304	20	304	304	20	304	304	20	304	304	20
	62	62	22	62	62	10	59	59	19	58	58	20
	3,685	3,685	591	3,653	3,653	590	3 532	3,532	495	3,460	3,460	522
	8,989	8,989	527	8,914	8,914	542	8,617	8,617	312	8,443	8,443	412
	2,372	2,372	7	2,348	2,348	71	2,270	2,270	2	2,224	2,224	30
	28	28	0	28	28	0	28	28	0	28	28	0
	35,958	35,958	10,834	35,595	35,595	10,103	34,597	34,597	9,356	34,079	34,079	9,544
	20,658	2[illegible]58	836	20,455	20,455	828	19,881	19,88[illegible]	444	19,585	19,585	693
	1,799	,810	0	1,781	1,781	44	1,731	1,731	18	1,705	1,705	40
	5,789	5,789	1,244	5,930	5,930	1,496	5,732	5,732	1,123	5,617	5,617	1,162
	3,137	3,137	0	[illegible],214	3,206	166	3,107	3,107	0	3,044	3,044	34
	229	229	0	234	234	13	227	227	1	222	222	3

OBJECTIVE: MINIMIZE COST (MEASURED BY RELATIVE PRIORITIES) OF BEING OVER/UNDER FINAL USER SUPPORT REQUIREMENTS AND OF RECRUITING/REDUCTIONS OF CIVILIAN MANPOWER

SUBJECT TO CONSTRAINTS OF:

TOTAL AMOUNT EACH FINAL USER SUPPORTED − POSSIBLE AMOUNT OVER + POSSIBLE AMOUNT UNDER = EACH FINAL USER SUPPORT REQUIREMENT

− TOTAL AMOUNT EACH FINAL USER SUPPORTED + SUM OF (PROPORTION OF OUTPUT EACH PRODUCER PROVIDES TO EACH FINAL USER X TOTAL OUTPUT OF EACH PRODUCER) = 0

TOTAL OUTPUT OF EACH PRODUCER ≤ BUDGET OF EACH PRODUCER

SUM OF (CIVILIAN MANPOWER REQUIRED FOR EACH UNIT OF SUPPORT X TOTAL OUTPUT OF EACH PRODUCER) − REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER = 0

REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER ≤ TOTAL CIVILIAN MANPOWER AVAILABLE

CIVILIAN MANPOWER ON-BOARD AT START = INITIAL POPULATION

− REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER FROM PREVIOUS PERIODS + CIVILIAN MANPOWER ON-BOARD AT PRESENT PERIOD − REQUIRED CIVILIAN MANPOWER PROVIDED BY NEW HIRES + EXCESS CIVILIAN MANPOWER = 0

Figure 3. DYNAMIC MULTI-LEVEL MODEL STRUCTURE

The remainder of the model structure is concerned with the various goal requirements and resource constraints which must be considered while trying to minimize goal discrepancies. The first group of conditions is concerned with setting the goals. This is accomplished by setting up an equation for each final user for each time period. This equation states that the total amount of support furnished each user less the amount over plus the amount under will be equal to the total requirement, or goal. In any solution one will obtain for each equation the level of output associated with the goal and either the overage or underage (or zero deviation if the goal is met right on). This stems from the fact that one cannot be both over and under a goal at the same time. It should be noted at this point that all outputs are expressed in their dollar equivalents so that one can be consistent in adding the outputs for each of the producers.

The next set of conditions ensures that the distribution of output from each of the producers is in the right proportion to the requirements of the final users. One can simultaneously calculate the support-on-support requirements, or alternatively, as is done here, a second stage allocation process may be used by means of the definitional relations. The latter is accomplished by using data from an input-output table which indicate for each producer the proportion of output required to support the final users. This ensures that the final users

will be supported at the level required in the model solution and at the same time obtain the total amount of each output which should be produced. These relationships are built into the model by specifying an equation for each final user. This equation states that the sum of the output of each producer multiplied by the proportion consumed by each final user minus the total amount each final user consumes from all producers will equal zero. This forces all the individual pieces of output from each type of support to be in the right proportion to the total.

The next section of the model bounds the amount of each kind of output to be produced. An equation is assigned to each producer for each time period which prescribes a limit to the amount that can be produced. This ensures that no producer will exceed its budget in trying to meet the overall goal. At the same time if an excess budget has been allocated to the producer it will show up in a non-zero value for the corresponding slack variable.

The remainder of the model consists of a modified version of the OCMM career management model. In this particular model structure no provision is made for the manpower goals. Thus, the manpower levels are not guided to conform with predetermined manpower requirements. The input-output and career management sub-models are coupled by means of equations which relate the amount of manpower required by each producer for a given level of final user demand.

In the numerical example four producers and two final users were included. Table 4 contains the base data for this example. The first four rows expressed in millions of dollars indicate the amount of output servives each of the using sectors consumes. (For example, Producer 1 provides 95 million dollars of service to itself, 120 million dollars to Producer 2, etc.) The last row contains the amount of manpower required by each of the producers (e.g., Producer 1 requires 60,000 men, and so on). Neither Final User 1 nor Final User 2 has any manpower associated with it since this model is oriented towards obtaining support establishment requirements to meet final user demand.

The next step is to convert these base data into utilization rates for use in the input-output formulation. As far as the different types of output are concerned, this is done for each user including producers (to obtain support-on-support requirements later) by dividing the amount consumed by each user by the total amount produced. For example, Producer 1 consumes 95/920 or 10.33 percent of its own output, Producer 2 consumed 120/920 or 13.04 percent of Producer 1's output, etc. The full array of these output usage rates is contained in Table 5. The manpower usage rates are obtained by dividing the total manpower of a producer by the total amount produced. Thus, in this example, dividing the 60,000 men of Producer 1 by 920 million dollars yields 65.22 men per million dollars.

Three alternatives will be developed to show the reaction of the model to changing data conditions. In all these examples the final user requirements will be assumed to be constant. However, it will be assumed that there will be 5 percent inflation in wage and related manpower costs in Period 1 and 4 percent in Period 2. The inflation rate is included by decreasing the manpower usage rates by the appropriate factor. In this way less manpower per million dollars of output is received. One should also be sure to increase the amount of final user demand in dollars to compensate for the fact that less output per dollar expended is obtained.

The non-constant input data for the three alternatives are given in Table 6. The goals and budgetary levels of the producers are not in strict proportion to the historical distribution of outputs. The goals in the second period are both lower and in a different proportion to the first period goals. Considering this, and the effects of inflation, one would expect not only that the producer budget levels will decline but also that the required manpower will decrease at an even greater rate.

The data in Table 7 are given on two categories of manpower—White Collar and Blue Collar. The manpower rates for each of the producers per million dollars of output are given and include the inflation factors previously mentioned. Additionally, a transition matrix is given which describes the internal movement and attrition from the work force.

TABLE 4

DYNAMIC MULTI-LEVEL MODEL
HISTORICAL USAGE DATA

	Producer 1	Producer 2	Producer 3	Producer 4	Final User 1	Final User 2	Total
Producer 1	95	120	80	200	125	300	920 Million Dollars
Producer 2	400	950	320	80	1,400	1,000	4,150
Producer 3	100	150	75	250	1,050	400	2,025
Producer 4	1,000	250	400	2,000	800	1,000	5,450
TOTAL	1,595	1,470	875	2,530	3,375	2,700	12,545
Civilian Manpower	60,000	12,000	23,000	200,000			295,000 Men

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 10.

TABLE 5

DYNAMIC MULTI-LEVEL MODEL
HISTORICAL USAGE DATA

	Producer 1	Producer 2	Producer 3	Producer 4	Final User 1	Final User 2
Producer 1	.1033	.1304	.0890	.2174	.1359	.3260
Producer 2	.0964	.2289	.0771	.0193	.3373	.2410
Producer 3	.0494	.0741	.0370	.1235	.5185	.1975
Producer 4	.1835	.0459	.0734	.3669	.1468	.1835
Civilian Manpower Per Million Dollars Output	65.22	2.89	11.36	36.70		

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 10.

TABLE 6

DYNAMIC MULTI-LEVEL MODEL
ALTERNATIVE INPUT DATA

Final User Requirements Assumed Constant for all Alternatives;
5% Inflation in Period 1, 4% Inflation in Period 2

ALTERNATIVE 1: Heavy Budget Cuts
ALTERNATIVE 2: No Budget Increases
ALTERNATIVE 3: Substantial Budget Increases

MANPOWER PER MILLION DOLLARS OUTPUT

	Producer 1	Producer 2	Producer 3	Producer 4	Remarks
Period 1	61.88	2.75	10.79	34.87	5% Inflation
Period 2	59.40	2.64	10.36	33.47	Additional 4% Inflation

FINAL USER SUPPORT REQUIREMENTS

	Period 1	Period 2
Final User 1	3,375	3,200
Final User 2	2,700	2,600

PRODUCER BUDGETARY AND MANPOWER LEVELS

	ALTERNATIVE 1		ALTERNATIVE 2		ALTERNATIVE 3	
	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
Producer 1	875	830	920	920	1,200	1,200
Producer 2	3,940	3,740	4,150	4,150	4,500	4,500
Producer 3	1,925	1,830	2,025	2,025	2,500	2,500
Producer 4	5,175	4,920	5,450	5,450	6,000	6,000
Civilian Manpower	295	295	295	295	295	295

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 14.

The data were arranged in the linear programming matrix given in Figure 4. The relative priorities on the final user goal discrepancies were set to equal one. A second set of relative priorities was included for the new hire and excess manpower columns of the model. These were needed to ensure that the model will first choose on-board manpower before hiring or releasing manpower. These relative priorities were set to a value of two.

The solution data are given in Table 8 through Table 10. In all of the alternatives, the model is sensitive to the manpower data. This is due to the weighting in the model objective to favor retention of on-board manpower. The most interesting example is the third alternative. Here, the model has not only balanced the manpower in each period but also has balanced the manpower over the two periods. Because of this fact, the total manpower and total budget for the overall system solution is less than the second

TABLE 7

DYNAMIC MULTI-LEVEL MODEL MANPOWER DATA

	Producer 1	Producer 2	Producer 3	Producer 4
BASE PERIOD ON-BOARD				
White	40,000	10,000	18,000	60,000
Blue	20,000	2,000	5,000	140,000
Total	60,000	12,000	23,000	200,000
MANPOWER PER MILLION DOLLARS OUTPUT				
White	43.48	2.41	8.89	11.01
Blue	21.74	0.48	2.47	25.67
5% Inflation One Period Later				
White	41.48	2.29	8.44	10.46
Blue	20.40	0.46	2.35	24.41
5% Inflation One Period Later; 4% Inflation Two Periods Later				
White	39.82	2.20	8.10	10.04
Blue	19.58	0.44	2.26	23.43

TRANSITION RATES (Read Down)

	White	Blue
White	.90	.05
Blue		.80

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 15.

alternative. This is particularly true of the manpower, where the total strength is 11,000 less in both the first and second periods when compared with the second alternative solution. Since the final user demands are the same in both the second and third alternative, this is a very satisfying result.

Table 9 gives the detailed manpower data for the three alternatives. It should be remembered that the transition rates are hypothetical. For example, excess White Collar manpower is obtained in all cases from Producer 4. This is the result of the large on-board complement of Blue Collar manpower in Producer 4. Since a five percent transfer rate of Blue Collar into White Collar was assumed, this results in 5,000 to 6,000 transferring each period. Because of this fact, the model in considering all of the constraints must generate excess White Collar personnel. In the real situation one would expect a lower transition rate of Blue Collar manpower into White Collar. These detailed manpower solution data nevertheless clearly demonstrate that the transition rates operate at least qualitatively in a manner which corresponds to what such a model design should produce.

	POSITIVE GOAL DISCREP.	NEGATIVE GOAL DISCREP.	FINAL USER SUPPORT	PRODUCER OUTPUT	ON-BOARD MANPOWER	NEW HIRE MAN POWER	EXCESS MAN POWER	SIGN	RHS
RELATIVE PRIORITIES	α	α				β	β		
FINAL USER GOALS	-1	1	1					=	FINAL USER SUPPORT REQUIREMENTS
	-1	1	1					=	
SUPPORT USAGE RATES			-1	I-O RATES				=	0
			-1	I-O RATES				=	0
PRODUCTION LIMITS				1				$\leq$	PRODUCER BUDGETARY LIMITS
				1				$\leq$	
PRODUCER OUTPUT/ MANPOWER RATES				$(m/\$)_1$	-1			=	0
				$(m/\$)_2$	-1			=	0
TOTAL MANPOWER CONSTRAINTS					1111			$\leq$	TOTAL MANPOWER AVAILABLE
					1111			$\leq$	
MANPOWER ATTRITION					1			=	INIT. POP.
					-M 1	-1	1	=	0
					-M 1	-1	1	=	0

Figure 4. DYNAMIC MULTI-LEVEL MODEL
LINEAR PROGRAMMING MATRIX

TABLE 8

DYNAMIC MULTI-LEVEL MODEL SOLUTION DATA

	Alternative 1			Alternative 2			Alternative 3		
	REQ.	ACT.	DIFF.	REQ.	ACT.	DIFF.	REQ.	ACT.	DIFF.
Period 1									
F.U. 1	3,375	3,206	-169	3,375	3,375		3,375	3,375	
F.U. 2	2,700	2,565	-135	2,700	2,700		2,700	2,700	
Period 2									
F.U. 1	3,200	3,045	-155	3,200	3,200		3,200	3,200	
F.U. 2	2,600	2,436	-164	2,600	2,600		2,600	2,600	

SUPPORT ELEMENTS

PRODUCER BUDGETARY AND MANPOWER RESOURCES

	Alternative 1			Alternative 2			Alternative 3		
	Available	Used	Unused	Available	Used	Unused	Available	Used	Unused
Period 1									
Producer 1	875	875		920	920		1,200	892	308
Producer 2	3,940	3,940		4,150	4,150		4,500	4,500	
Producer 3	1,925	1,925		2,025	2,025		2,500	1,876	624
Producer 4	5,175	5,175		5,450	5,450		6,000	5,200	800
Manpower	295	264	31	295	278	17	295	267	28
Period 1									
Producer 1	830	830		920	886	34	1,200	859	341
Producer 2	3,740	3,740		4,150	4,150		4,500	4,500	
Producer 3	1,830	1,830		2,025	1,751	274	2,500	1,601	899
Producer 4	4,920	4,920		5,450	5,258	192	6,000	5,007	993
Manpower	295	241	54	295	256	39	292	260	50

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 23.

Support-on-support requirements as shown in Table 10 can easily be generated multiplying the resultant budget levels by the appropriate rows of the input-output matrix. For example, in Alternative 2, the resultant budget levels of 920 for Producer 1 in Period 1 is multiplied by 0.1304 to obtain the 120 million dollars of support which Producer 1 must provide to Producer 2.

Some of the potential management uses of the multi-level model are:

1. Ways for evaluating the impact on manpower and other resource requirements of additions to or deletions from final user support requirements.

TABLE 9

DYNAMIC MULTI-LEVEL MODEL SOLUTION DATA

MANPOWER DATA

		Alternative 1			Alternative 2			Alternative 3		
		On-Board	New Hire	Excess	On-Board	New Hire	Excess	On-Board	New Hire	Excess
Period 1	White	36.3		0.7	38.2	1.2		37.1		
Producer 1	Blue	17.9	1.9		18.8	2.8		18.2	2.2	
	White	9.0		0.1	9.5	0.4		10.3	1.2	
Producer 2	Blue	1.8	0.2		1.9	0.3		2.1	0.5	
	White	16.2		0.2	17.1	0.6		15.8		0.6
Producer 3	Blue	2.2		1.8	2.3		1.7	2.1		1.9
	White	54.1		6.9	57.0		4.0	54.4		6.6
Producer 4	Blue	126.3	14.3		133.0	21.0		126.9	14.9	
Period 2	White	33.1		0.5	35.2			34.2		
Producer 1	Blue	16.3	2.0		17.3	2.3		16.8	2.3	
	White	8.2			19.1	0.5		9.9	0.5	
Producer 2	Blue	1.6	0.2		1.8	0.3		2.0	0.3	
	White	14.8	0.1		14.2			13.0		1.4
Producer 3	Blue	1.9	0.2		1.9	0.1		1.7		
	White	49.4		5.6	52.8		5.2	50.3		5.0
Producer 4	Blue	115.3	14.2		123.2	16.8		117.3	15.8	

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 24.

2. The provision of an explicitly delineated structure for making resource allocation decisions and observing potential discrepancies.

3. Systematically supplied ways for evaluating inconsistencies between manpower and budgetary allocation decisions.

4. Systematic ways for evaluating effects of inflation or other such changes in operating force support requirements and manpower requirements.

5. Determination and depiction of the effects of attrition and internal manpower transfers on both short and long run decisions.

6. The integration of career management, training, recruitment, and advancement planning with budgetary and strategic decisions.

TABLE 10

DYNAMIC MULTI-LEVEL MODEL

SUPPORT-ON-SUPPORT REQUIREMENTS

Alternative 1					
Producer	Period	Producer 1	Producer 2	Producer 3	Producer 4
1	1	90	114	76	190
2	1	380	902	304	76
3	1	95	143	71	238
4	1	950	238	380	1,899
1	2	86	108	72	180
2	2	361	856	288	72
3	2	90	136	68	226
4	2	903	226	361	1,658

Alternative 2					
Producer	Period	Producer 1	Producer 2	Producer 3	Producer 4
1	1	95	120	80	200
2	1	400	950	320	80
3	1	100	150	75	250
4	1	1,000	250	400	2,000
1	2	92	116	77	193
2	2	400	950	320	80
3	2	86	130	65	216
4	2	965	241	386	1,929

Alternative 3					
Producer	Period	Producer 1	Producer 2	Producer 3	Producer 4
1	1	92	116	78	194
2	1	434	1,030	347	87
3	1	93	139	69	232
4	1	954	239	382	1,908
1	2	89	112	75	187
2	2	434	1,030	347	87
3	2	70	119	59	198
4	2	919	230	368	1,837

Source: Charnes, Cooper, Niehaus, and Sholtz, "Multi-Level Models," p. 25.

CHAPTER III
SOFTWARE SYSTEMS DESIGN

The purpose of this chapter is to provide the strategic and tactical considerations involved in the implementation of multi-level model applications software. This will be done through an in-depth discussion of past and current OCMM manpower models applications software designs. This software system has been named the Five Year Navy Civilian Manpower Plan (FYNCIMP) system. It is part of the Navy Automated Civilian Manpower Information System (NACMIS).

The FYNCIMP system has been developed from its very beginning through the use of a research oriented point of view. Thus, as it has progressed, large portions of the experimental prototypes have been transferred to an operational status. The two largest of the currently operational subsystems include the Transition Rate subsystem used to calculate personnel movements data and the Expected Retirement subsystem used to develop projected data on retirement.

Experimental portions of the FYNCIMP system include a specialized linear programming matrix generator and report writer for manpower planning. This capability, which is still under development, has been designed to provide the ability to generate a wide variety of outputs of a nature that can be easily understood by non-mathematical professionals. An exploratory development effort supported by the Office of Naval Research is also under way to investigate the conversational use of the models by means of remote computer terminals. The purpose of this research is to explore and test a variety of ways in which managers can be led to perceptions that will enable them to take advantage of the OCMM models and the computer arrangements they provide.

Transition Rate Subsystem

One of the early strategic design decisions was the development of the Transition Rate subsystem. This decision was reached even before the first models were fully developed. The need for personnel movements data became apparent in the early model development sessions. One of the ideas which was felt worth pursuing was the extension of Markov models into a dynamic framework. Based upon the numerical examples published by Vroom and MacCrimmon,[1] it was decided to develop a small operational prototype. This prototype consisted of one computer program utilizing OCMM's Personnel Automated Data System (PADS).[2] The test was restricted to the inter-grade movement of GS-830 Mechanical Engineers. This test population was small enough to be processed easily and large enough to ensure statistically significant results. Figure 5 is an example of the output from this initial prototype. The results from this research gave an indication that it was possible to produce meaningful data upon which to build a modeling framework.

For sake of clarity and compactness, a hypothetical example will now be given of the basic data processing steps used to develop the transition data. For purposes of this illustration the following job categories will be used:

[1]V. Vroom and K. MacCrimmon, "Toward a Stochastic Model of Managerial Career Development: Research Proposal," *Administrative Science Quarterly*, June, 1968.

[2]PADS provided the base for the extension of the Navy's civilian personnel file into the NACMIS system.

MARKOV CHAIN PROGRAM
WRITTEN JUNE 1967 R.J. NIEHAUS OCMM

RELATIVE FREQUENCY OF MOVEMENT 0830 GRADES

February 28, 1967 — April 30, 1967

PERIOD T GRADE	PERIOD T+1. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	EXIT	NUMBER AT T
1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	28.	.	14.	.	.	.	.	.	.	.	.	.	.	.	1.	45.
6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	141.	.	16.	.	.	.	.	.	.	.	.	.	4.	161.
8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
9	.	.	.	.	.	.	1.	.	202.	.	29.	2.	.	.	.	.	.	.	10.	244.
10	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	631.	37.	.	.	.	.	.	.	26.	694.
12	.	.	.	.	.	.	.	.	.	.	1.	766.	18.	.	.	.	.	.	20.	805.
13	.	.	.	.	.	.	.	.	.	.	.	2.	394.	1.	.	.	.	.	6.	405.
14	.	.	.	.	.	.	.	.	.	.	.	.	.	83.	.	.	.	.	1.	84.
15	.	.	.	.	.	.	.	.	.	.	.	.	.	1.	22.	.	.	.	.	23.
16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1.	.	.	.	1.
17	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Figure 5. MARKOV CHAIN PROGRAM, JUNE 1967

18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
ORIGINAL GROUP IN EACH STATE AT T+1	.	.	.	.	28.	.	156.	.	218.	.	661.	807.	414.	85.	22.	1.	.	.
INPUT REMAINING AT T+1	.	.	1.	.	16.	.	41.	.	40.	.	72.	132.	192.	51.	18.	1.	.	.
ON BOARD AT T+1	.	.	1.	.	44.	.	197.	.	258.	.	733.	939.	606.	136.	40.	2.	.	.

TOTAL AT PERIOD T 2392.

TOTAL INPUT REMAINING AT T+1 564.

TOTAL ON BOARD AT T+1 2956.

Figure 5--Continued

MARKOV CHAIN PROGRAM, JUNE 1967

RELATIVE FREQUENCY OF MOVEMENT 0830 GRADES

PERIOD T February 28, 1967 PERIOD T+1, April 30, 1967

PERIOD GRADE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	EXIT	NUMBER AT T
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
5	.000	.000	.000	.000	.622	.000	.311	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.067	
6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
7	.000	.000	.000	.000	.000	.000	.876	.000	.099	.000	.000	.000	.000	.000	.000	.000	.000	.000	.025	
8	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
9	.000	.000	.000	.000	.000	.000	.004	.000	.828	.000	.119	.008	.000	.000	.000	.000	.000	.000	.041	
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
11	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.909	.053	.000	.000	.000	.000	.000	.000	.037	
12	.000	.000	.000	.000	.000	.000	.000	.000	.000	.030	.861	.052	.022	.000	.000	.000	.000	.000	.035	

Figure 5--Continued

13	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.005	.978	.002	.000	.000	.000	.000	.015
14	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.988	.000	.000	.000	.000	.012
15	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.043	.957	.000	.000	.000	.000
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	1.000	.000	.000	.000
17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
18	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
ORIGINAL GROUP, IN EACH STATE AT T+1	.	.	.	.	28.	.	156.	.	218.	.	661.	807.	414.	85.	22.	1.	.	.	
INPUT REMAINING AT T+1	.	.	1.	.	16.	.	41.	.	61.	.	72.	132.	192.	31.	18.	1.	.	.	
ON BOARD AT T+1	.	.	1.	.	44.	.	197.	.	258.	.	733.	939.	6[illegible]	136.	40.	2.	.	.	

TOTAL AT PERIOD T	2462.
TOTAL INPUT REMAINING AT T+1	564.
TOTAL ON BOARD AT T+1	2956.

Figure 5–Continued

Category	Code
Management	MGT
General Administration	GEN
Skilled Worker	SW
Unskilled Worker	UW

The initial data collection procedure consists of putting together on one magnetic tape an employee's personnel record from two points in time. This will later provide the ability of essentially taking a "snapshot" of the manpower population between two time periods. The input data file is similar to that shown in Figure 6.

EMPLOYEE NUMBER	JOB CATEGORY	
	JUNE 1970	JUNE 1971
3024	MGT	MGT
3025	SW	
3047	GEN	MGT
3072		UW

Figure 6. DATA COLLECTION PROCEDURE

The transition rate computer program is used to develop a comparison of the two time periods on one report. In the numerical example as shown in Figure 7 it can be noted that of the 50 employees in the Management category in June, 1970, only 40 remained in that category by June, 1971. Five of the 50 had transferred to the General Administrative category and five had left the population. By totaling the columns, the on-board total for June, 1971 can be obtained. To obtain the rates of movement, the number in each category of a given row is divided by the row total. These rates are given in Figure 8.

Job Category	Totals June 1970	MGT	GEN	UW	SW	EXITS
MGT	50	40	5			5
GEN	300	10	210			80
UW	600			360	60	180
SW	500				450	50
ENTRIES		5	110	300		
Totals June 1971		55	325	660	510	

Figure 7. HISTORICAL TRANSITION STATISTICS

	MGT	GEN	UW	SW	EXITS
MGT	.80	.10			.10
GEN	.03	.70			.27
UW			.60	.10	.30
SW				.90	.10

Figure 8. TRANSITION RATES

The success of the small prototype invited the development of a more generalized capability.[1] Specifications were developed which included the requirements for matrices which displayed the following movements:

1. between grade levels
2. between occupational groups
3. between individual occupations
4. between activities or installations
5. between geographic areas

The computer system was further generalized by the provision of an extract capability to limit the population to be considered. Additionally, a research capability was provided to assist in capturing the relevant population to be studied. The flow diagram for the Transition Rate subsystem is given in Figure 9.

In addition to the extract capability, the Transition Rate subsystem includes the option to limit the population under consideration within the computer programs themselves. The variables upon which the limits could be applied in the original version included: sex, birthdate, service computation data, bureau or major command, activity or installation, occupation, and grade or level. In the current version data can also be produced with limits on: minority group, geographic area, Program Element of the Five Year Defense Program, Major Appropriation, and various occupation-grade level aggregations. Examples of outputs might include the inter-occupation transitions of women within a given major command. Similarly, grade transitions of personnel born since 1935 and within a given occupation could be developed.

Both a matrix and a listing output are available with the Transition Rate program. The matrix output is limited to 30 or less categories. The listing output can accommodate up to 500 categories. The listing output includes considerable English text to facilitate understanding of the data. Examples of these outputs are shown as Figures 10 and 11.

The transition matrices can be used initially to derive an accurate quantified idea of the actual personnel movements. At the same time as the matrix is produced, a list of entries to and losses from the population can be produced. These provide a trail from which further researches can be made. These entry and loss lists are divided into intra-Navy and extra-Navy reports. Examples of these lists are given in Figures 12 through 15.

It was decided to program the generalized transition rate system in COBOL. The need for efficient file handling and management report writing far outweighed the mathematical processing requirements. Since the original version of the system was programmed, computer programming techniques have been devised to speed up the processing of matrix algebra operations.

The development of the Transition Rate subsystem illustrates some of the strategic and tactical considerations of computer software development to support models. The initial commitment was to a small prototype. Data from this small prototype was used to determine what capabilities should be included in the more generalized system. The generalized system then went through a series of major and minor modifications. Currently, there have been three major versions of the system which have incorporated over twenty minor changes. The major changes were preceded by the need to incorporate the Transition Rate subsystem into a larger data system to support the models. These major changes also included ideas from extensive data studies accomplished by major users of the system outputs.

Initial Model Prototypes

Paralleling the development of the Transition Rate subsystem was the initial and continuing construction of the mathematical models. The initial goal programming model mathematics provided for the

[1] A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "Measurement of Personnel Movement" *The Journal of Navy Civilian Manpower Management*, Vol. III, No. 1, March, 1969.

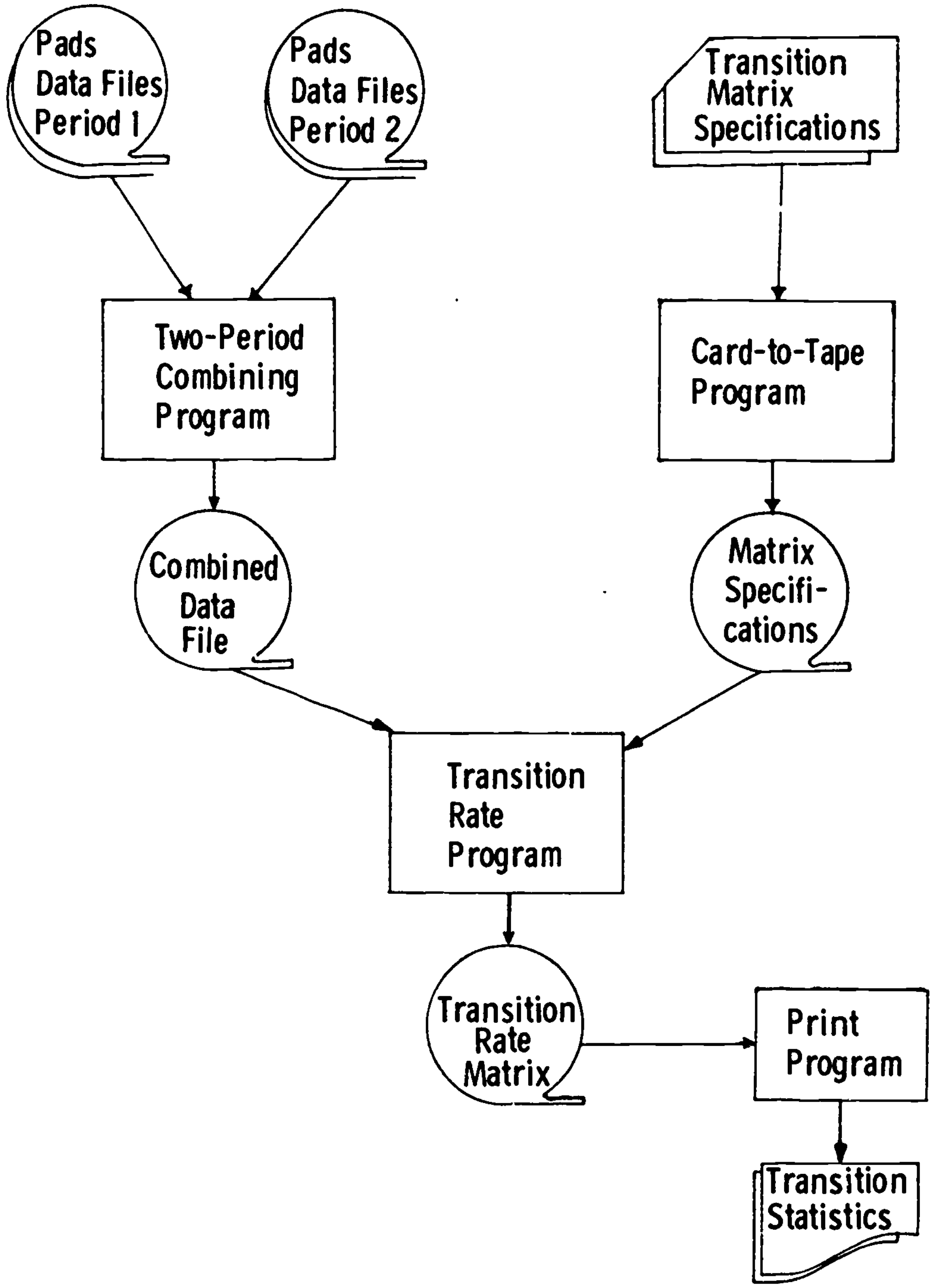

Figure 9. TRANSITION RATE SUBSYSTEM

specification of an "optimum" set of net manpower requirements.[1] These net requirements were used then in an algorithm to develop the number of personnel needed to be recruited in each period of the forecast. Constraints on the net manpower goals included the transition rates, mean salary of each job category, funding available to hire additional personnel, and the relative priority of each job category.

[1] A. Charnes, W. W. Cooper, and R. J. Niehaus, "A Goal Programming Model for Manpower Planning," In John Blood, Ed., *Management Science in Planning and Control* (New York: Technical Association of the Pulp and Paper Industry, 1968).

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                          :CUPATION MOVEMENT BY U.S. CITIZE
                                FROM JUN 70 TO JUN 71
                              PROCUREMENT CAREER PROGRAM

SEX     BIRTHDATE      SERVICE     MAJOR           NAVAL   PROGRAM   SNDL   UIC
                       COMP DATE   CLAIM   ROCMM   DIST    ELEMENT   ACT    ACT         GRADE
BOTH    1 -- 1         1 -- 1      ALL     ALL     ALL     ALL       ALL    ALL         ALL

OF THE    382 EMPLOYEES IN OCCUPATION 01101 AT JUN 70:

     322 OR  84.3 % STAYED IN 01101          3 OR  0.8 % WENT TO 01102
       2 OR   0.5 % WENT TO 01103
      11 OR   2.9 % STAYED IN THE NAVY BUT LEFT THIS POPULATION.          44 OR  11.5 % LEFT THE NAVY.

OF THE  2,383 EMPLOYEES IN OCCUPATION 01102 AT JUN 70:

       2 OR   0.1 % WENT TO 01101         2054 OR 86.2 % STAYED IN 01102
      41 OR   1.7 % STAYED IN THE NAVY BUT LEFT THIS POPULATION.         284 OR  11.9 % LEFT THE NAVY.

OF THE    110 EMPLOYEES IN OCCUPATION 01103 AT JUN 70:

      87 OR  79.1 % STAYED IN 01103          1 OR  0.9 % WENT TO 01150
       4 OR   3.6 % STAYED IN THE NAVY BUT LEFT THIS POPULATION.          18 OR  16.4 % LEFT THE NAVY.

OF THE    756 EMPLOYEES IN OCCUPATION 01150 AT JUN 70:

       4 OR   0.5 % WENT TO 01101            2 OR  0.3 % WENT TO 01102
       3 OR   0.4 % WENT TO 01103          616 OR 81.5 % STAYED IN 01150
      46 OR   6.1 % STAYED IN THE NAVY BUT LEFT THIS POPULATION.          85 OR  11.2 % LEFT THE NAVY.

   OTHER ENTRIES FROM INSIDE NAVY     ENTRIES FROM OUTSIDE OF NAVY     TOTAL ON BOARD INCLUDING
    BETWEEN JUN 70 AND JUN 71:         BETWEEN JUN 70 AND JUN 71:    INTERNAL TRANSFERS AT JUN 71:

          37 INTO 01101                     44 INTO 01101                   410 IN 01101
          88 INTO 01102                     92 INTO 01102                 2,240 IN 01102
          12 INTO 01103                      4 INTO 01103                   108 IN 01103
          33 INTO 01150                     25 INTO 01150                   675 IN 01150

TOTAL AT JUN 70 IS   3,631.   TOTAL AT JUN 71 IS   3,433.
* * * * * * * * * * * * * * * *
* * * * * * * * * * * * * * * *
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Figure 10. TRANSITION RATE SUBSYSTEM, LISTING REPORT

RELATIVE ¿QUENCY OF MOVEMENT INTER-OCCUPAT

FROM JUN 70 TO JUN 71

SEX BOTH — BIRTHDATE 000 TO 000 — SERVICE COMPUTATION DATE 000 TO 000 — MAJOR CLAIMANT — ACTIVITY — GRADE LEVEL

OCCUPATION	NO AT JUN70	01101	01102	01103	01150	EXITS
01101	382	.043	.008	.005		.144
01102	2383	.001	.862			.136
01103	11C			.791	.009	.200
01150	756	.005	.003	.004	.015	.173
: GAINS		37	88	12	33	
E GAINS		44	92	4	25	
NO AT JUN71		410	2240	108	675	

TOTAL AT JUN 70 IS 3,631 TOTAL AT JUN 71 IS 3,433

Figure 11. TRANSITION RATE SUBSYSTEM, MATRIX REPORT

INT[illegible] NAVY ENTRIES BETWEEN JUN 70 AND J[illegible] 71

SEX	BIRTHDATE	SERVCOMP	MAJCL	BNDL ACT	OCCUPATION	GRADE	SALARY	STEP	MOVEMENT
M	27 MAY 45	22 JAN 68	70 19	1451808 1440810	GS 00230 GS 01101	09 07	PA 09881 PA 10584	01 08	INTRA-NAVY ENTRY
F	24 FEB 43	02 MAY 64	37 37	9099037 9099037	GS 00301 GS 01101	09 09	PA 09881 PA 10819	01 02	INTRA-NAVY ENTRY
M	25 MAY 46	25 MAR 67	17 17	4603915 4603915	GS 00301 GS 01102	09 11	PA 09881 PA 13036	01 02	INTRA-NAVY ENTRY
M	19 SEP 39	22 SEP 65	24 24	5862920 5862920	GS 00301 GS 01102	09 11	PA 09881 PA 12615	01 01	INTRA-NAVY ENTRY
F	01 OCT 19	10 FEB 44	78 78	4577950 4577950	GS 00301 GS 01102	06 07	PA 09481 PA 10584	10 08	INTRA-NAVY ENTRY
F	11 AUG 47	15 SEP 69	17 17	4603915 4603915	GS 00301 GS 01102	07 07	PA 08098 PA 08868	01 02	INTRA-NAVY ENTRY
M	20 FEB 32	09 NOV 61	09 1[illegible]	9090009 1681900	GS 00301 GS 01102	09 09	PA 10210 PA 11168	02 03	INTRA-NAVY ENTRY
F	24 MAR 25	13 FEB 61	25 25	3070115 3070055	GS 00301 GS 01102	05 07	PA 07202 PA 09440	04 04	INTRA-NAVY ENTRY
M	03 APR 46	20 NOV 68	19 19	1461900 6802960	GS 00301 GS 01102	07 09	PA 08098 PA 10470	01 01	INTRA-NAVY ENTRY
F	27 MAY 47	02 SEP 69	24 24	5862920 5862920	GS 00301 GS 01102	07 09	PA 08098 PA 10470	01 01	INTRA-NAVY ENTRY
F	09 OCT 22	03 AUG 53	25 25	3070130 3070130	GS 00301 GS 01102	04 07	PA 07218 PA 09154	08 03	INTRA-NAVY ENTRY
F	17 JAN 33	18 NOV 65	25 25	3070130 3070130	GS 00301 GS 01102	04 05	PA 06243 PA 07169	03 02	INTRA-NAVY ENTRY
F	30 AUG 48	08 JUN 68	24 24	5835955 5835955	GS 00301 GS 01102	04 05	PA 06048 PA 06938	02 01	INTRA-NAVY ENTRY
F	15 MAR 38	26 APR 59	70 25	3139010 3070745	GS 00301 GS 01102	03 07	PA 05212 PA 09154	01 03	INTRA-NAVY ENTRY
F	27 AUG 22	21 DEC 59	25 25	2506600 2506600	GS 00301 GS 01150	05 07	PA 07638 PA 08582	06 01	INTRA-NAVY ENTRY
M	09 JAN 38	25 MAR 59	19 19	1440810 1440810	GS 00305 GS 01101	03 07	PA 06778 PA 09440	10 04	INTRA-NAVY ENTRY
F	18 OCT 26	23 MAR 69	19 19	1462750 1462750	GS 00312 GS 01102	05 07	PA 06766 PA 08582	02 01	INTRA-NAVY ENTRY
F	09 SEP 37	10 FEB 69	25 25	3070130 3070130	GS 00312 GS 01102	04 05	PA 06048 PA 07169	02 02	INTRA-NAVY ENTRY
F	08 NOV 47	25 DEC 66	14 14	5580700 5580700	GS 00318 GS 01102	06 07	PA 07294 PA 08582	01 01	INTRA-NAVY ENTRY
F	13 JAN 23	16 APR 63	27 27	8373200 8373200	GS 00318 GS 01102	05 05	PA 08074 PA 08555	08 08	INTRA-NAVY ENTRY
F	10 FEB 24	24 OCT 60	23 22	2943400 6373496	GS 00318 GS 01102	05 09	PA 09988 PA 10819	10 02	INTRA-NAVY ENTRY
F	14 JUL 19	05 AUG 52	25 25	3070775 3070745	GS 00318 GS 01102	05 07	PA 08510 PA 09726	10 05	INTRA-NAVY ENTRY
F	23 FEB 19	29 JUN 47	78 30	1625275 6042275	GS 00318 GS 01102	05 06	PA 08510 PA 10049	10 10	INTRA-NAVY ENTRY
F	29 APR 39	03 SEP 61	25 25	2547420 2547420	GS 00322 GS 01102	04 05	PA 06828 PA 07862	06 05	INTRA-NAVY ENTRY
F	17 AUG 38	29 NOV 67	24 25	6152675 2547422	GS 00322 GS 01102	03 05	PA 05560 PA 07169	03 02	INTRA-NAVY ENTRY
F	06 JUL 32	27 JAN 69	17 25	3148200 3070745	GS 00322 GS 01102	03 05	PA 05908 PA 07400	05 03	INTRA-NAVY ENTRY
F	07 APR 47	22 JUL 69	19 14	1461900 5580700	GS 00334 GS 01101	05 07	PA 06548 PA 08868	01 02	INTRA-NAVY ENTRY
M	21 AUG 35	29 MAR 66	70 19	1451808 1440810	GS 00335 GS 01101	07 11	PA 09988 PA 12615	08 01	INTRA-NAVY ENTRY
M	21 OCT 18	05 JAN 47	19 19	1440065 1440065	GS 00343 GS 01101	11 12	PA 13890 PA 16543	06 04	INTRA-NAVY ENTRY
M	30 JUL 23	27 APR 43	19 19	1440065 1440065	GS 00343 GS 01101	11 12	PA 13890 PA 16543	06 04	INTRA-NAVY ENTRY
M	13 MAY 21	11 JAN 46	19 19	1440810 1440810	GS 00343 GS 01101	09 11	PA 10868 PA 12615	04 01	INTRA-NAVY ENTRY
M	25 MAY 17	27 MAR 56	19 19	4312500 4312500	GS 00343 GS 01102	11 11	PA 13096 PA 13878	04 04	INTRA-NAVY ENTRY
M	03 MAY 29	10 JAN 47	19 19	1440440 1440440	GS 00344 GS 01101	09 11	PA 11855 PA 13457	07 03	INTRA-NAVY ENTRY
M	20 DEC 20	09 DEC 45	19 19	1440440 1440440	GS 00344 GS 01101	12 12	PA 11138 PA 17545	03 06	INTRA-NAVY ENTRY
M	16 JUN 33	17 JUN 55	19 19	1440440 1440440	GS 00344 GS 01101	09 09	PA 10868 PA 11517	04 04	INTRA-NAVY ENTRY
M	22 FEB 24	11 AUG 42	19 19	1440440 1440440	GS 00344 GS 01101	09 09	PA 12513 PA 13611	09 10	INTRA-NAVY ENTRY
M	05 OCT 22	09 OCT 43	19 19	1440440 1440440	GS 00344 GS 01101	11 11	PA 13493 PA 14720	05 06	INTRA-NAVY ENTRY
M	28 APR 18	09 JUN 41	19 19	1440735 1440735	GS 00344 GS 01101	07 09	PA 10528 PA 11866	10 05	INTRA-NAVY ENTRY
M	28 SEP 20	09 OCT 47	19 19	1440440 1440440	GS 00344 GS 01101	11 11	PA 13493 PA 15141	05 07	INTRA-NAVY ENTRY
F	21 AUG 15	04 JAN 56	30 30	4806770 4806770	GS 00344 GS 01102	07 07	PA 09718 PA 10298	07 07	INTRA-NAVY ENTRY
M	03 SEP 31	11 JUN 53	37 37	9099037 9099037	GS 00345 GS 01101	14 15	PA 20298 PA 25059	02 02	INTRA-NAVY ENTRY
F	21 APR 41	28 JUN 66	24 24	5862920 5835900	GS 00345 GS 01102	05 07	PA 07420 PA 08582	05 01	INTRA-NAVY ENTRY
M	10 MAY 34	21 JUL 52	24 24	5862920 5862920	GS 00345 GS 01150	12 13	PA 14192 PA 17761	01 01	INTRA-NAVY ENTRY
F	01 MAR 22	28 OCT 55	60 60	1450202 1450202	GS 00385 GS 01101	05 07	PA 08510 PA 09726	10 05	INTRA-NAVY ENTRY
F	09 JAN 09	15 AUG 42	33 24	4222500 6152600	GS 00501 GS 01102	07 07	PA 09988 PA 08868	08 02	INTRA-NAVY ENTRY
M	15 APR 08	22 FEB 63	60 60	1450202 1450202	GS 00570 GS 01101	04 04	PA 06438 PA 06823	04 04	INTRA-NAVY ENTRY
F	18 JAN 23	05 SEP 57	30 30	4806770 4806770	GS 00520 GS 01101	05 05	PA 08074 PA 08786	08 09	INTRA-NAVY ENTRY
M	06 JUL 45	15 NOV 66	19 19	1440810 1440810	GS 00525 GS 01101	05 07	PA 06548 PA 08582	01 01	INTRA-NAVY ENTRY
F	20 JUL 46	17 JUN 68	24 24	5835900 5835900	GS 00560 GS 01102	07 07	PA 08098 PA 10584	01 08	INTRA-NAVY ENTRY
M	30 JUL 38	24 NOV 64	24 25	5867190 3070055	GS 00801 GS 01102	11 12	PA 14287 PA 16042	03 03	INTRA-NAVY ENTRY

Figure 12. TRANSITION RATE SUBSYSTEM, INTRA-NAVY ENTRY LIST

EXT: NAVY ENTRIES BETWEEN JUN 70 AND J 71

SEX	BIRTHDATE	SERVCOMP	MAJCL	SNDL ACT	OCCUPATION	GRADE	SALARY	STEP	MOVEMENT
M	18 OCT 21	29 DEC 41	24	5867905	GS 01101	15	PA 26675	04	ENTERED THE NAVY
M	17 FEB 13	21 APR 36	19	1440600	GS 01101	15	PA 24251	01	ENTERED THE NAVY
M	21 JAN 23	02 JUN 46	14	5580700	GS 01101	15	PA 29099	07	ENTERED THE NAVY
M	14 MAR 28	10 OCT 62	14	5580700	GS 01101	14	PA 20815	01	ENTERED THE NAVY
M	28 OCT 21	16 APR 53	17	4806760	GS 01101	14	PA 22897	04	ENTERED THE NAVY
M	30 OCT 26	29 SEP 68	24	6152600	GS 01101	14	PA 21509	02	ENTERED THE NAVY
M	03 NOV 23	17 NOV 42	19	1462525	GS 01101	11	PA 13878	04	ENTERED THE NAVY
M	05 SEP 32	02 APR 58	60	1450772	GS 01101	11	PA 12615	01	ENTERED THE NAVY
M	07 DEC 21	18 APR 40	24	6152510	GS 01101	11	PA 13878	04	ENTERED THE NAVY
M	10 MAR 25	07 JUL 47	70	6030805	GS 01101	11	PA 12615	01	ENTERED THE NAVY
M	04 MAR 18	27 NOV 44	19	1462525	GS 01101	11	PA 14720	06	ENTERED THE NAVY
M	19 JUN 35	29 APR 58	30	4806770	GS 01101	11	PA 13036	02	ENTERED THE NAVY
M	14 NOV 23	04 NOV 42	19	1440440	GS 01101	11	PA 14299	05	ENTERED THE NAVY
M	07 MAY 19	14 FEB 66	62	1452616	GS 01101	11	PA 12615	01	ENTERED THE NAVY
M	15 MAR 24	000015	70	1451652	GS 01101	11	PA 13036	02	ENTERED THE NAVY
M	08 AUG 16	18 SEP 63	19	4312500	GS 01101	11	PA 12615	01	ENTERED THE NAVY
M	26 JUL 22	14 OCT '8	70	6030805	GS 01101	12	PA 15541	02	ENTERED THE NAVY
M	30 OCT 16	10 APR 55	70	6030695	GS 01101	12	PA 15040	01	ENTERED THE NAVY
M	21 JUN 26	03 NOV 46	70	1451885	GS 01101	12	PA 15040	01	ENTERED THE NAVY
M	22 JUN 21	17 FEB 65	23	2443290	GS 01101	09	PA 10470	01	ENTERED THE NAVY
M	06 AUG 22	17 MAR 42	19	1440735	GS 01101	09	PA 16543	10	ENTERED THE NAVY
M	25 JUL 26	21 JAN 55	19	1440440	GS 01101	09	PA 11866	05	ENTERED THE NAVY
M	19 NOV 29	24 AUG 66	25	2506600	GS 01101	09	PA 10470	01	ENTERED THE NAVY
M	04 JUL 17	13 JUN 50	24	5867625	GS 01101	06	PA 10049	10	ENTERED THE NAVY
M	25 MAR 27	27 FEB 60	27	8139875	GS 01101	07	PA 09726	05	ENTERED THE NAVY
M	11 AUG 29	14 AUG 52	19	1462525	GS 01101	05	PA 09017	10	ENTERED THE NAVY
M	21 JUL 26	07 JAN 61	23	6170750	GS 01101	05	PA 09017	10	ENTERED THE NAVY
M	31 MAY 41	22 MAR 67	19	1440810	GS 01101	07	PA 08582	01	ENTERED THE NAVY
M	03 OCT 12	21 NOV 63	17	4600120	GS 01101	07	PA 11156	10	ENTERED THE NAVY
M	25 JAN 39	21 JUN 60	19	1440810	GS 01101	09	PA 11866	05	ENTERED THE NAVY
M	02 AUG 41	07 NOV 61	19	1440910	GS 01101	09	PA 11866	05	ENTERED THE NAVY
M	28 DEC 47	05 MAY 69	19	1440810	GS 01101	09	PA 10470	01	ENTERED THE NAVY
M	01 DEC 44	04 SEP 63	19	1440910	GS 01101	09	PA 11517	04	ENTERED THE NAVY
M	22 AUG 18	05 JAN 38	78	6541500	GS 01101	13	PA 19537	04	ENTERED THE NAVY
M	26 AUG 21	18 DEC 60	19	6802960	GS 01101	13	PA 17761	01	ENTERED THE NAVY
M	09 DEC 13	24 DEC 44	19	1462525	GS 01101	13	PA 21313	07	ENTERED THE NAVY
M	09 NOV 20	23 MAY 59	22	6373496	GS 01101	12	PA 16042	03	ENTERED THE NAVY
M	17 FEB 32	09 FEB 58	30	4806770	GS 01101	12	PA 16543	04	ENTERED THE NAVY
F	01 MAR 17	23 DEC 61	70	1451160	GS 01101	10	PA 11517	01	ENTERED THE NAVY
M	23 FEB 36	13 MAY 69	62	1452200	GS 01101	09	PA 10470	01	ENTERED THE NAVY
M	20 MAY 14	29 JUN 43	70	1451885	GS 01101	10	PA 12285	03	ENTERED THE NAVY
M	24 DEC 06	10 APR 68	63	2476625	GS 01101	09	PA 10470	01	ENTERED THE NAVY
M	21 MAR 27	15 OCT 48	70	1451885	GS 01101	10	PA 11901	02	ENTERED THE NAVY
M	29 JUN 19	17 MAY 68	62	1452590	GS 01101	10	PA 11517	01	ENTERED THE NAVY
M	13 OCT 26	25 AUG 59	23	5129925	GS 01102	13	PA 20129	05	ENTERED THE NAVY
M	30 APR 27	04 NOV 65	27	9099027	GS 01102	11	PA 13036	02	ENTERED THE NAVY
M	19 DEC 21	03 APR 45	70	6175785	GS 01102	11	PA 14720	06	ENTERED THE NAVY
M	10 SEP 42	13 MAY 68	24	6152516	GS 01102	11	PA 12615	01	ENTERED THE NAVY
M	20 SEP 22	30 DEC 66	24	6152600	GS 01102	11	PA 15983	09	ENTERED THE NAVY
F	29 MAR 30	22 NOV 52	25	3070745	GS 01102	11	PA 12615	01	ENTERED THE NAVY

Figure 13. TRANSITION RATE SUBSYSTEM, EXTRA-NAVY ENTRY LIST

IN. -NAVY LOSSES BETWEEN JUN 70 AND J. 71

SEX	BIRTHDATE	SERVCOMP	MAJCL	SNDL ACT	OCCUPATION	GRADE	SALARY	STEP	MOVEMENT
M	19 OCT 16	26 DEC 43	17 17	4603915 4603915	GS 01101 GS 00301	14 14	PA 21608 PA 23591	04 05	INTRA-NAVY EXIT
M	14 FEB 28	01 DEC 57	19 19	1440775 1440775	GS 01101 GS 00344	07 07	PA 08908 PA 09440	04 04	INTRA-NAVY EXIT
M	15 OCT 38	17 OCT 61	19 19	1440810 1462750	GS 01101 GS 00345	11 12	PA 12699 PA 15040	03 01	INTRA-NAVY EXIT
F	08 JAN 40	17 JUN 59	19 19	1440810 1462750	GS 01101 GS 00345	11 12	PA 11905 PA 15040	01 01	INTRA-NAVY EXIT
M	18 NOV 34	17 FEB 56	19 19	1440810 1462750	GS 01101 GS 00345	11 12	PA 13096 PA 15040	04 01	INTRA-NAVY EXIT
M	18 JUL 30	09 MAR 69	70 22	7126700 7125700	GS 01101 GS 01001	11 11	PA 11905 PA 13457	01 03	INTRA-NAVY EXIT
F	26 JUL 28	18 JUN 46	23 23	6170700 6170700	GS 01101 GS 01107	07 07	PA 09448 PA 10298	06 07	INTRA-NAVY EXIT
M	25 JAN 27	23 OCT 45	19 19	1440065 1440065	GS 01101 GS 01152	11 11	PA 14287 PA 15562	07 08	INTRA-NAVY EXIT
M	06 APR 11	29 JUL 41	19 19	1440735 1440735	GS 01101 GS 01601	15 15	PA 24411 PA 26675	03 04	INTRA-NAVY EXIT
M	23 FEB 13	09 JUN 47	23 23	2943400 2943400	GS 01101 GS 01670	12 11	PA 17976 PA 19048	09 10	INTRA-NAVY EXIT
M	04 MAY 38	22 JUL 57	24 24	5862920 5862920	GS 01101 GS 01670	12 12	PA 14192 PA 15541	01 02	INTRA-NAVY EXIT
M	23 MAY 39	14 SEP 63	14 37	5580210 4115900	GS 01102 GS 00201	11 09	PA 12302 PA 13262	02 09	INTRA-NAVY EXIT
M	30 MAY 25	28 JUL 43	23 23	1746600 1746600	GS 01102 GS 00301	14 15	PA 23573 PA 26675	07 04	INTRA-NAVY EXIT
M	28 OCT 38	20 NOV 64	24 39	5862920 2925905	GS 01102 GS 00301	11 12	PA 11905 PA 15040	01 01	INTRA-NAVY EXIT
F	13 AUG 27	21 AUG 61	24 24	5867650 5867650	GS 01102 GS 00301	07 04	PA 09098 PA 08945	01 10	INTRA-NAVY EXIT
F	16 JAN 34	000016	27 27	9099027 9099027	GS 01102 GS 00301	07 05	PA 08638 PA 09017	03 10	INTRA-NAVY EXIT
F	30 JUL 11	09 SEP 51	25 25	2506200 2506200	GS 01102 GS 00301	05 05	PA 08074 PA 08555	08 08	INTRA-NAVY EXIT
F	14 AUG 36	14 MAY 65	27 27	8373200 8373200	GS 01102 GS 00301	05 05	PA 07202 PA 07631	04 04	INTRA-NAVY EXIT
F	11 JUL 44	20 SEP 65	19 37	1461900 4115900	GS 01102 GS 00301	09 11	PA 09881 PA 12615	01 01	INTRA-NAVY EXIT
M	14 FEB 34	29 JUL 57	17 17	4603915 4603915	GS 01102 GS 00301	13 13	PA 17319 PA 20129	02 05	INTRA-NAVY EXIT
F	29 JUL 23	03 MAY 42	17 37	4603915 4115900	GS 01102 GS 00301	13 13	PA 18437 PA 19537	04 04	INTRA-NAVY EXIT
F	10 MAR 20	27 FEB 49	78 78	6480750 6480750	GS 01102 GS 00312	07 05	PA 09988 PA 09017	08 10	INTRA-NAVY EXIT
M	13 SEP 46	18 JUN 68	17 17	4600120 4600120	GS 01102 GS 00334	07 09	PA 08098 PA 10470	01 01	INTRA-NAVY EXIT
M	07 AUG 27	24 AUG 49	37 37	9099037 9099037	GS 01102 GS 00345	13 13	PA 17878 PA 19537	03 04	INTRA-NAVY EXIT
M	13 JUN 28	13 MAY 50	23 23	6170750 6170750	GS 01102 GS 00505	12 12	PA 16084 PA 17044	05 05	INTRA-NAVY EXIT
M	24 FEB 43	29 FEB 65	19 19	1389666 1389666	GS 01102 GS 00510	09 07	PA 10539 PA 10298	03 07	INTRA-NAVY EXIT
M	21 SEP 17	27 FEB 61	24 24	6152294 6152294	GS 01102 GS 00510	13 13	PA 18437 PA 20129	04 05	INTRA-NAVY EXIT
F	03 DEC 42	06 OCT 64	24 24	6152700 5867250	GS 01102 GS 00525	05 05	PA 06984 PA 07631	03 04	INTRA-NAVY EXIT
F	07 SEP 18	02 NOV 49	22 22	6373700 6373700	GS 01102 GS 00560	07 07	PA 08098 PA 08868	01 02	INTRA-NAVY EXIT
M	13 MAR 27	11 SEP 60	19 19	1462750 6802960	GS 01102 GS 00560	13 13	PA 18437 PA 20129	04 05	INTRA-NAVY EXIT
M	11 OCT 26	24 MAY 62	25 25	2547475 2547475	GS 01102 GS 00810	12 11	PA 15611 PA 16825	04 10	INTRA-NAVY EXIT
F	22 JAN 24	06 MAY 45	24 24	5867600 5867600	GS 01102 GS 01105	07 06	PA 08908 PA 09791	04 09	INTRA-NAVY EXIT
M	03 JUN 19	15 DEC 43	60 60	6029705 6029705	GS 01102 GS 01105	07 07	PA 09178 PA 10012	05 06	INTRA-NAVY EXIT
F	26 FEB 14	22 AUG 55	23 23	6170595 6170595	GS 01102 GS 01105	07 07	PA 09178 PA 10012	05 06	INTRA-NAVY EXIT
M	03 MAY 37	29 MAR 66	62 62	1452292 1452292	GS 01102 GS 01105	06 07	PA 09023 PA 09154	04 03	INTRA-NAVY EXIT
F	16 SEP 17	24 AUG 55	23 23	6170595 6170595	GS 01102 GS 01105	07 07	PA 08908 PA 09726	04 05	INTRA-NAVY EXIT
F	30 MAR 31	15 NOV 53	78 78	5857600 5857600	GS 01102 GS 01105	07 08	PA 09448 PA 10757	06 05	INTRA-NAVY EXIT
F	27 JAN 15	02 MAY 51	14 14	3860930 3860930	GS 01102 GS 01105	09 09	PA 10539 PA 11517	03 04	INTRA-NAVY EXIT
F	07 OCT 23	26 OCT 47	24 24	5867625 5867625	GS 01102 GS 01105	09 08	PA 11197 PA 12021	05 09	INTRA-NAVY EXIT
F	07 NOV 13	22 OCT 48	78 24	6480750 5867600	GS 01102 GS 01105	11 06	PA 12699 PA 11434	03 10	INTRA-NAVY EXIT
F	21 SEP 28	17 FEB 50	25 25	3070950 3070950	GS 01102 GS 01106	07 07	PA 09718 PA 10298	07 07	INTRA-NAVY EXIT
F	27 SEP 25	04 OCT 65	23 23	6170200 6170200	GS 01102 GS 01106	07 05	PA 08098 PA 08786	01 09	INTRA-NAVY EXIT
F	07 AUG 33	01 JUL 52	78 78	6480750 6430750	GS 01102 GS 01106	09 04	PA 11526 PA 08065	06 10	INTRA-NAVY EXIT
M	05 AUG 11	12 DEC 38	24 24	6152650 6152650	GS 01102 GS 01152	11 11	PA 12699 PA 13878	03 04	INTRA-NAVY EXIT
M	05 DEC 12	14 NOV 43	78 78	4577950 4577950	GS 01102 GS 02001	09 06	PA 11855 PA 12337	07 10	INTRA-NAVY EXIT
M	04 AUG 24	08 DEC 56	23 60	6170595 2391500	GS 01102 GS 02001	05 07	PA 07420 PA 08582	05 01	INTRA-NAVY EXIT
F	30 OCT 24	10 JAN 45	24 24	5867650 5867650	GS 01102 GS 02005	07 06	PA 08908 PA 10049	04 10	INTRA-NAVY EXIT
F	31 JUL 31	14 MAY 52	27 27	8270175 8270175	GS 01102 GS 02005	05 06	PA 08074 PA 09017	08 06	INTRA-NAVY EXIT
M	08 MAR 16	08 DEC 64	27 27	8373605 8373605	GS 01102 GS 02010	07 07	PA 08638 PA 09440	03 04	INTRA-NAVY EXIT
M	06 FEB 41	19 AUG 61	23 23	6170200 6170200	GS 01102 GS 02010	09 07	PA 10210 PA 10870	02 09	INTRA-NAVY EXIT

Figure 14. TRANSITION RATE SUBSYSTEM, INTRA-NAVY LOSS LIST

EX -NAVY LOSSES BETWEEN JUN 70 AND J 71

SEX	BIRTHDATE	SERVCOMP	MAJCL	SNDL ACT	OCCUPATION	GRADE	SALARY	STEP	MOVEMENT
M	28 DEC 14	21 AUG 56	25	2547460	GS 01102	08	PA 08956	01	LEFT THE NAVY
M	24 OCT 23	11 OCT 49	78	5857600	GS 01102	08	PA 11348	09	LEFT THE NAVY
F	10 OCT 22	25 JUN 53	24	5867625	GS 01102	04	PA 06828	06	LEFT THE NAVY
F	17 MAR 47	03 JAN 67	19	1682333	GS 01102	05	PA 06766	02	LEFT THE NAVY
M	19 MAR 46	17 JUN 68	17	4603915	GS 01102	05	PA 06548	01	LEFT THE NAVY
F	21 APR 11	01 APR 53	27	8373200	GS 01102	05	PA 07202	04	LEFT THE NAVY
M	01 DEC 47	09 JUN 69	24	5862920	GS 01102	05	PA 06549	01	LEFT THE NAVY
F	22 JUN 47	18 JUN 69	19	1461900	GS 01102	05	PA 06548	01	LEFT THE NAVY
F	21 JUL 11	11 SEP 40	23	6170595	GS 01102	05	PA 08510	10	LEFT THE NAVY
F	23 DEC 45	05 JUL 66	17	4603915	GS 01102	09	PA 09881	01	LEFT THE NAVY
F	13 APR 44	09 AUG 66	23	6170595	GS 01102	09	PA 10210	02	LEFT THE NAVY
M	26 MAY 38	18 NOV 58	39	2925905	GS 01102	09	PA 09881	01	LEFT THE NAVY
F	10 JUN 21	11 MAR 41	23	5864500	GS 01102	09	PA 11855	07	LEFT THE NAVY
F	16 JUL 15	29 SEP 43	23	5864500	GS 01102	09	PA 12842	10	LEFT THE NAVY
F	12 OCT 16	18 MAY 51	17	6303602	GS 01102	09	PA 10868	04	LEFT THE NAVY
M	04 APR 15	09 JUL 40	78	6797500	GS 01102	09	PA 11197	05	LEFT THE NAVY
M	09 SEP 10	23 MAR 45	27	8373605	GS 01102	09	PA 11855	07	LEFT THE NAVY
M	17 MAY 24	05 OCT 64	19	4805736	GS 01102	09	PA 10868	04	LEFT THE NAVY
F	12 SEP 18	31 MAR 42	23	5510650	GS 01102	09	PA 11855	07	LEFT THE NAVY
M	30 DEC 42	12 AUG 62	23	1746600	GS 01102	09	PA 10210	02	LEFT THE NAVY
F	18 MAY 16	06 FEB 35	23	5129270	GS 01102	09	PA 11526	06	LEFT THE NAVY
M	30 JUN 26	10 AUG 45	11	2782800	GS 01102	09	PA 10868	04	LEFT THE NAVY
F	24 JUN 05	16 NOV 43	23	6170575	GS 01102	09	PA 10868	04	LEFT THE NAVY
M	07 OCT 12	06 DEC 43	23	2943400	GS 01102	09	PA 11197	05	LEFT THE NAVY
M	05 MAR 11	07 OCT 41	23	5864500	GS 01102	09	PA 12184	08	LEFT THE NAVY
F	24 APR 19	23 JUN 41	23	5510650	GS 01102	09	PA 12513	09	LEFT THE NAVY
M	16 AUG 15	19 MAR 44	17	4600110	GS 01102	09	PA 11855	07	LEFT THE NAVY
M	06 MAR 46	03 JUN 68	17	4603915	GS 01102	09	PA 09881	01	LEFT THE NAVY
M	11 MAR 11	25 NOV 40	23	1746600	GS 01102	09	PA 12184	08	LEFT THE NAVY
F	07 JAN 46	06 SEP 68	23	5510600	GS 01102	09	PA 09881	01	LEFT THE NAVY
F	27 MAR 02	05 JUL 41	23	6170200	GS 01102	09	PA 11197	05	LEFT THE NAVY
M	02 JUL 20	27 SEP 42	17	4600110	GS 01102	09	PA 10868	04	LEFT THE NAVY
M	17 APR 10	25 SEP 41	65	4495365	GS 01102	09	PA 11197	05	LEFT THE NAVY
F	17 OCT 14	01 OCT 40	24	5867651	GS 01102	09	PA 11855	07	LEFT THE NAVY
M	11 MAY 24	30 AUG 64	19	4805736	GS 01102	09	PA 10539	03	LEFT THE NAVY
F	01 OCT 17	07 SEP 66	23	2943400	GS 01102	09	PA 10539	03	LEFT THE NAVY
M	25 JUL 39	14 NOV 66	25	5119800	GS 01102	09	PA 10210	02	LEFT THE NAVY
M	28 APR 23	14 MAY 47	25	3070120	GS 01102	09	PA 11197	05	LEFT THE NAVY
M	03 MAR 45	29 APR 68	19	1461900	GS 01102	09	PA 09881	01	LEFT THE NAVY
F	07 MAR 33	01 JAN 56	78	6450750	GS 01102	09	PA 10868	04	LEFT THE NAVY
F	19 MAY 18	02 FEB 42	19	1339666	GS 01102	09	PA 11855	07	LEFT THE NAVY
M	03 FEB 41	12 JUN 62	23	1746600	GS 01102	09	PA 10868	04	LEFT THE NAVY
F	25 MAR 10	07 JUN 36	23	5129925	GS 01102	13	PA 18996	05	LEFT THE NAVY
M	18 OCT 11	21 AUG 40	14	5590540	GS 01102	13	PA 19555	06	LEFT THE NAVY
M	22 MAR 16	29 JUL 40	19	1461900	GS 01102	13	PA 19555	06	LEFT THE NAVY
M	01 MAR 06	26 JAN 48	23	6190925	GS 01102	13	PA 19555	06	LEFT THE NAVY
M	21 JAN 11	08 JUL 40	19	4805300	GS 01102	13	PA 20114	07	LEFT THE NAVY
M	10 NOV 11	13 FEB 41	23	1746600	GS 01102	13	PA 20114	07	LEFT THE NAVY
M	16 SEP 25	25 SEP 44	23	5864500	GS 01102	13	PA 18437	04	LEFT THE NAVY
M	08 NOV 15	30 SEP 35	24	5862920	GS 01102	13	PA 18996	05	LEFT THE NAVY

Figure 15. TRANSITION RATE SUBSYSTEM, EXTRA-NAVY LOSS LIST

As a first step in moving the goal programming model mathematics toward implementation, a "toy" problem using hypothetical data was constructed.[1] This example consisted of the solution of a linear program covering four job categories for two periods. It contained only fourteen equations and was solved in seconds of computer time. This small size is emphasized to point out that large problems and long computer runs are not always needed to test model concepts. The small size also facilitates checkout since the influence of a given variable is obvious for a given set of data.

The results of the toy problem provided an insight into the system design for a larger prototype. The system design concept shown in Figure 16 was the first step in this extension. The idea was to have four main subsystems: (1) the Transition Rate subsystem, (2) a linear program data assembler or matrix generator, (3) commercially available linear programming software, and (4) a management reports writer. The system would give technical reports to those responsible for the operation of the model and would give management reports to the users of the model.

As stated earlier, this version of the goal programming model requires net manpower requirements as a data input. In this case, net requirements are defined as the difference between overall or gross manpower requirements and the residual population on-board from some base point in time. Through the use of the transition rates it is possible to obtain a projection of the original population in future time periods. This can be shown by continuing with the hypothetical example used to illustrate the development of the transition rates. To obtain the residual population remaining one year later, the starting population is multiplied by the transition rates as shown in Figure 17. The projected totals for each category are obtained by adding up the columns for each category. These new totals can then be used to make the projection for the next year. This process of "rolling the totals" through the transition rates is continued until the required number of years has been projected. These data can then be subtracted from the set of gross requirements data to obtain the net requirements.

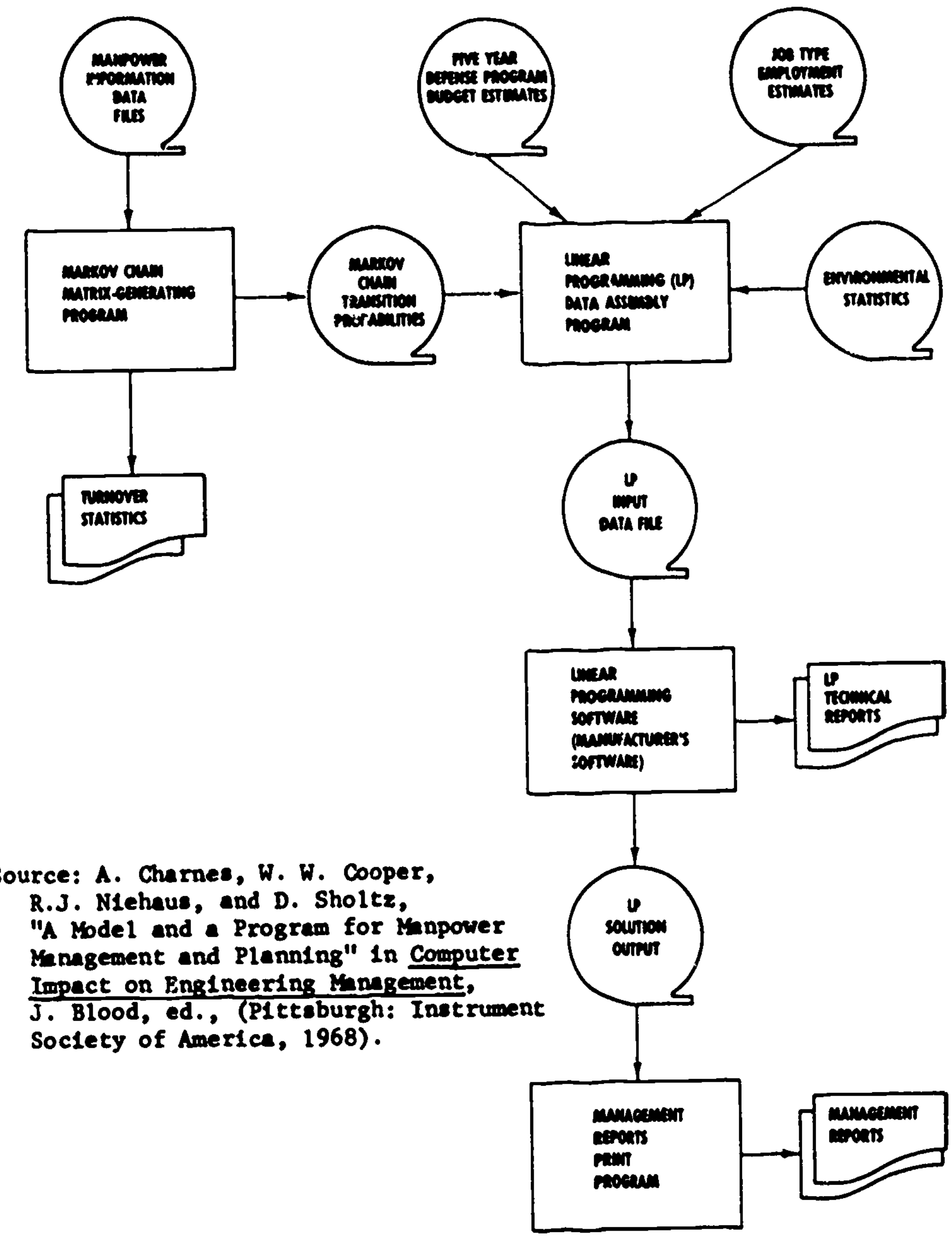
The method of obtaining net requirements as outlined above was incorporated into a small scale prototype. It was felt that this small scale prototype was needed since a full scale system required a large commitment of resources. Also, it was felt that many computer programming problems would most likely appear. Thus, the operational test had to be large enough to ferret out the computer software problems but small enough to keep the associated data handling manageable. Further, this study had to allow a focus on the management information to be developed as well as the computer software concepts to be tested. With these ideas in mind, it was decided that the prototype study would encompass a population of approximately 3,000 General Schedule personnel by occupational-grade level grouping categories.[2]

At this point interest in the research was expressed by the Programming Division of the Naval Facilities Engineering Command. With their cooperation in providing the gross requirements data, the computer software was programmed and the net requirements data obtained. This test required approximately three man-months of programming and testing. As a result, the needed design information for the software improvements required for a large-scale operational system was obtained. More importantly, the management implications of the data became apparent. Some of these included:[3]

- Net requirements can provide a basis for planning the input of personnel to command-wide professional training programs.

- The fact that excess people are projected for some of the categories is indicated by negative numbers. Probably a management decision must be made either to change the gross requirements or

[1] For the solution to this example, see A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "A Model and a Program for Manpower Management and Planning," *Computer Impact on Engineering Management*, Joint Engineering Management Conference, Philadelphia, September 30-October 1, 1968 (Pittsburgh: Instrument Society of America, 1968).
[2] For a description of this prototype study and its extension into the Five Year Navy Civilian Manpower Plan (FYNCIMP) system, see A. Charnes, W. W. Cooper, R. J. Niehaus, and W. N. Price, "Application of Computer-Assisted Techniques to Manpower Planning," *The Journal of Navy Civilian Manpower Management*, September, 1969.
[3] *Ibid.*



Source: A. Charnes, W. W. Cooper, R.J. Niehaus, and D. Sholtz, "A Model and a Program for Manpower Management and Planning" in Computer Impact on Engineering Management, J. Blood, ed., (Pittsburgh: Instrument Society of America, 1968).

Figure 16. MANPOWER PLANNING MODEL SYSTEM FLOW CHART

	June 1972			
	MGT	GEN	UW	SW
MGT	44	6		
GEN	10	228		
UW			396	66
SW				459
Totals	54	234	396	525

Figure 17. PROJECTION OF PRESENT POPULATION

shift personnel from the categories indicated. Re-examination should also be made of the historical movement rates used to make the projections.

- The data can also be used to study ways to shift personnel in order to obtain the right balance by occupational categories and grade levels. This most likely would require additional techniques and tests.

Integration of Research with Implementation

Other research continued while the prototype computer software was being programmed and tested. This included a number of large-scale tests of the Transition Rate subsystem. Included were studies involving Navywide, command, geographic area, and individual installation movement data. As a result, this capability was beginning to be used for supporting operational data requirements. Additionally, the mathematical research was concentrated on the development of a suitable algorithm for projecting retirements. This was important since many Navy civilians were close to becoming eligible for retirement.

The idea which was used for projecting the number expected to retire rests on the assumption that those within five years of retirement eligibility will not change job categories. Using historical data one can compute the rates at which those who are eligible will actually retire. These rates are then multiplied by those eligible in the first year of the forecast. This results in a projection of (a) those who will retire, and (b) those who will remain in the organization. For the second year the process is repeated using those who are first eligible in the second year plus those remaining from those first eligible in the first year. These calculations can be repeated in a similar manner for each of the years used in the overall projection.

Modification of the goal programming model mathematics was made based upon the differences in the net requirements as a result of the retirement projection. An algorithm was also developed to obtain recruiting requirements from the set of "optimum" net requirements provided by the model solutions. This was necessary as the original model did not provide the number of personnel to be recruited year by year. These revised mathematics along with the results from the prototype test with Naval Facilities Command data were used to develop the specifications for the first comprehensive version of the Five Year Navy Civilian Manpower Plan (FYNCIMP) system. The revised mathematics, including a "live" numerical example, were summarized in a paper by Charnes, Cooper, Niehaus, and Sholtz.[1] This paper was presented at the NATO Conference "Mathematical Models for the Management of Manpower Systems," on September 1-5, 1969 at Porto, Portugal.

An important issue which was addressed in the implementation process was the system for coding the job categories. Current computer processing limitations dictated that no more than 500 job categories be included. In addition to these technical limitations, there is a need for a coding system which is consistent with management practices within the Navy and data available on the labor supplies, outside the Navy. The Civil Service Commission occupational coding system was not suitable since it was designed primarily for position classification rather than manpower planning purposes. Based upon these considerations, Treires[2] of the Office of Civilian Manpower Management devised a new and improved occupational coding system.

The chief feature of the FYNCIMP coding system is that it has been designed to be consistent to the greatest extent possible with the U. S. Census occupational groups. As a result it provides the capability (not yet used) of relating the Navy internal labor demand with the external national supply. To the knowledge of the researcher, this is the only occupational coding system for internal use within a government agency which includes this feature. The coding system aggregates the approximately 425 U. S. Civil Service Commission General Schedule (White Collar) occupation codes and 18 grades into 100 occupation groups and 5 grade groupings. Similarly, the approximately 800 ungraded (Blue Collar) occupations and 50 or so levels have been aggreated into 100 occupation groups and 5 level groupings.

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[1]A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "A Model for Civilian Manpower Management and Planning in the U. S. Navy," in *Models of Manpower Systems*, A. R. Smith, Ed. (London: The English Universities Press, 1970).
[2]J. Treires, "Counting Heads in the Federal Service," *Personnel Administration*, November-December, 1971.

Continued research for new ideas is one of the strategic considerations to ensure an improved modeling system. This research must be directed in such a way, however, that it can reinforce the already implemented portions of the system. It is very easy to do research continually and never to provide much of the improved capability to potential users. In order to ensure that the OCMM modeling system maintained a strong research base and also began to deliver usable data products to users, a three-pronged attack was initiated in 1970. This included: (1) research into the integration of resource allocation models with the manpower models; (2) research into the computability of very large manpower models using goal programming; and (3) continuation of the implementation and testing of the FYNCIMP system.

Focus on the possible integration of the resource allocation models with the manpower models was started by means of a multi-stage systems concept. This concept is shown in Figure 18. It also includes the possible integration of assignment models for shorter term operational planning. This design contains three models which are staged such that the results of one stage are used to provide the constraints for the succeeding stage(s). The models are coupled by means of deterministic algorithms to provide additional data not directly provided in the model solutions. This multi-stage concept has given way to the multi-level concept explained in the previous chapter. Additionally, very recent research in the area of assignment models may provide a different approach to interrelating the various levels of manpower decision making.[1]

In 1970 mathematical research by Charnes, Cooper, and Niehaus[2] was initiated to include training possibilities in the model structures. This included dividing the workforce population into three groups: (1) those in the organization during the period who received training, (2) those in the organization who did not receive training, and (3) those recruited from the outside during the period. These data were put together by means of generalized network techniques. Other information which is considered is the kind and cost of the training for each of the periods under consideration. All of these data are then used in a goal programming model to obtain the best mix of training and recruiting to support the accomplishment of a given set of tasks. After comments were received at the Institute of Management Sciences meeting in London, the model was extended to include the ideas of chance-constrained programming.[3]

The interrelationships of the model mathematics and the testing with small prototypes can be illustrated with the generalized network model. One problem which was not found until the testing had been attempted was that the time phasing was out of synchronization. Some of the data were introduced as happening at the end of a period. Other data were introduced at the beginning of a period. The correction involved more care in describing wh[illegible] particular points in time. Similar synchronization problems had shown up earlier during t[illegible] NCIMP computer programs with various data files. These were resolved by the additio[illegible] check a time period indicator which was placed on all the data tapes. If the subroutine e[illegible] coded with a time period other than the one required, processing is terminated with an error messa[illegible]

The toy problems for the generalized network model also demonstrated the need for some basic structural changes in the model mathematics. It was found that the workforce population should have been divided into four and not three groups. There is a need to take account of those on-board in a succeeding period but not required. That is, one must take into account firing requirements as well as recruiting requirements. Once discovered, this was easy to repair since it involved setting up an additional set of categories for those who would have to be fired. Relative priorities had to be assigned in the objective function of the model for these new categories, as one normally would consider such action only as a last resort. It is interesting to note that the need for these changes was not caught either by the people at the London TIMS meeting or by the referees prior to publication of the model mathematics. This does not mean that the basic mathematical development

[1] A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "An Algorithm for Multi-Attribute Assignment Models and Spectral Analyses for Dynamic Organization Design," Research Report No. 78 (Austin: Center for Cybernetics Studies, University of Texas, 1972).

[2] A. Charnes, W. W. Cooper, and R. J. Niehaus, "A Generalized Network Model for Training and Recruiting Decisions in Manpower Planning," in *Manpower and Management Science*, A. R. Smith, Ed. (London: The English Universities Press, 1971).

[3] *Ibid.*

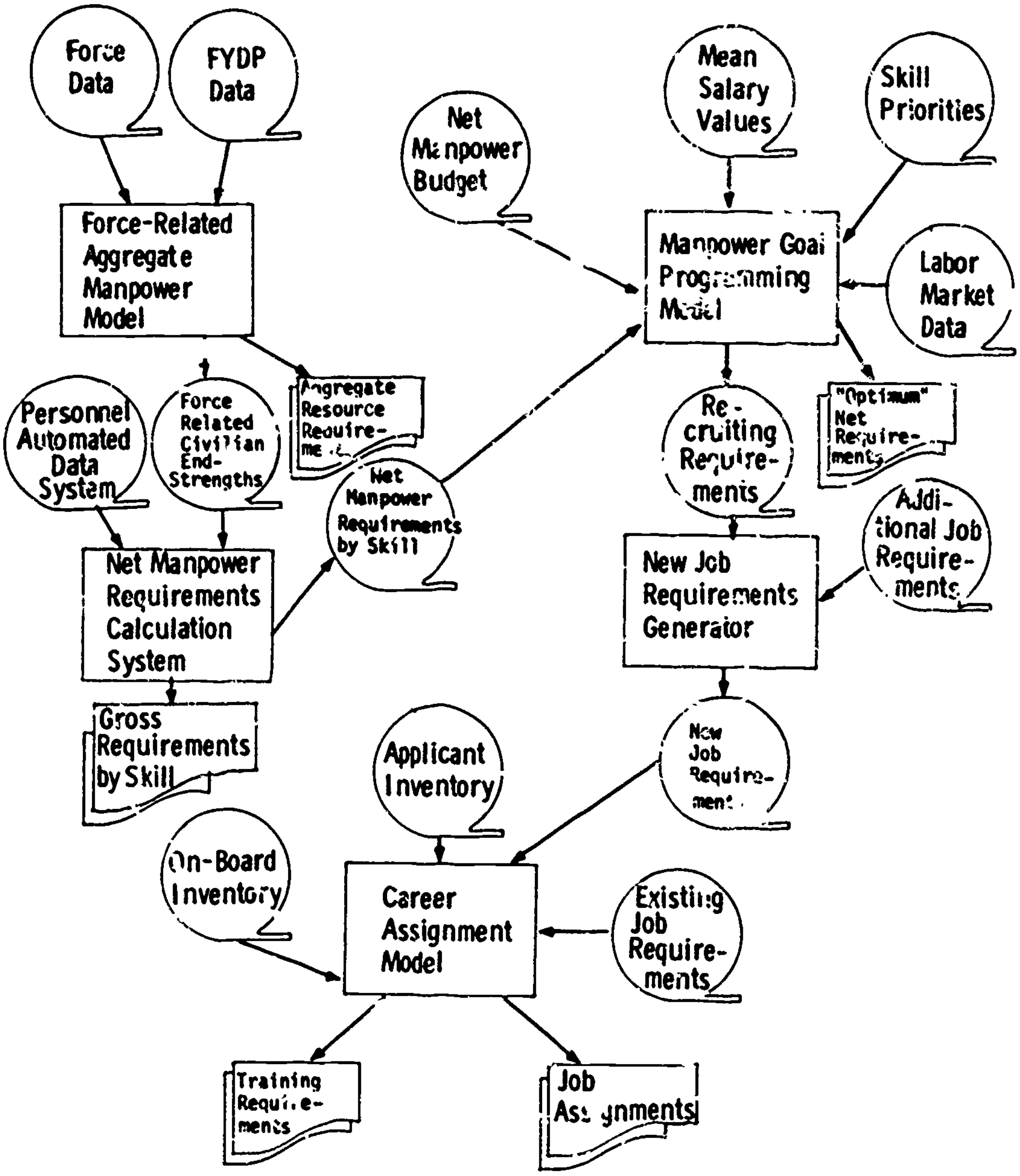

Figure 18. AN INTEGRATED MANPOWER PLANNING AND CAREER ASSIGNMENT DATA SYSTEM

process should be faulted, for it was pointed out in the technical report that the next step was the construction of numerical examples.[1]

A state-of-the-art improvement which the generalized network model provided is that it operates entirely with gross or total manpower requirements data. Studies made with the models which optimized a set of net requirements indicated that these net numbers have a tendency toward randomness except for large manpower populations. Thus, in a small manpower population (i.e., 1000-5000 people) the net requirements model are very difficult to use. This fact and the availability of the generalized model led to a major revision of the FYNCIMP system.

[1] Ibid.

The small cell size problem was also addressed for the portions of the FYNCIMP system which generated the expected retirements data. Empirical research indicated that actual retirement rates are not markedly different for different occupations within a major occupation group as defined by the Bureau of Labor Statistics (i.e., Professional and Technical, Managerial and Administrative, etc.). This led to a FYNCIMP design change in which the retirement rates are first developed for each of the major occupation groups. These rates are then used for all of the occupations within the appropriate major occupation group. Here, it was the computer processing techniques rather than the mathematical algorithm which was changed.

The multi-level model structure was developed through a series of experiments using toy problems. The objective of these experiments was to examine the possible combinations of the Center for Naval Analyses NARM model[1] and the OCMM models In this case the various model constructions were revised many times until the right combination was obtained. Eventually, the model structure discussed in the previous chapter was developed. This method of development illustrates that one need not start with the theoretical mathematics to obtain new extensions. This effort again points to the desirability of using small problems in the early model development phase.

In addition to research into new model constructions, attention was paid to the computability of large models (i.e., several thousand equations with several thousand variables). A research study was established which included the participation of J. Stutz at the University of Texas.[2] In this study the linear programming input data was developed at OCMM and transferred to the University of Texas for solution. The first two problems involved the ungraded population of one of the shipyards. The first problem was a two year projection and included 800 equations and 1,500 variables. It was solved in less than one minute on the CDC 6600 computer using the OPHELIE II linear programming code. The second problem was a five year projection and included 1,500 equations and 2,500 variables. It was solved in five minutes on the CDC 6600. The final problem involved a five year projection of the Navy's White Collar civilian workforce. This problem included 3,200 equations and 5,200 variables. It was solved in sixteen minutes on the CDC 6600. These results very conclusively indicate that it is practical to consider the implementation of such large-scale manpower planning models.

Five Year Navy Civilian Manpower Plan System

This chapter will be concluded with a general description of the current version of the Five Year Navy Civilian Manpower Plan (FYNCIMP) system. This version of the system includes the cumulative results of all of the design considerations which have been discussed. The FYNCIMP system has four main sections which are very much interrelated. The first section deals mainly with input data preparation, the second with retirement forecasting, the third with gross manpower requirements and transition rates generation, and the fourth with linear programming. As data flow through the system, intermediate results are printed as they have a management use in their own right. The system flow is given in Figure 19.

In the input data preparation section, data from OCMM's Personnel Automated Data System (PADS) and from the Navy Comptroller's Navy Cost Information System (NCIS) are merged to form the FYNCIMP combined population file. PADS is a file of approximately 340,000 records on the civilian employees of the Navy updated weekly via AUTODIN which is the Department of Defense worldwide data communications network. NCIS is the Navy's system for maintaining Five Year Defense Program (FYDP) data. In building the FYNCIMP combined population file two periods of PADS data along with identification data concerning the FYDP, organizational and geographical location data are combined with a new data element called "FYNCIMP state" for each employee. A state is that combination of data elements which will be used in making the personnel movement or transition calculations. The purpose of this is to code all the employee records with a systematic numerical code so that matrix computations can be made.

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[1] J. H. Augusta, R. A. Jenner, and G. W. Ryhanych, "Interim Input-Output Resource Allocation Model," Center for Naval Analysis Research Contribution, No. 134, 2 March 1970.

[2] For a summary of the results of this study see the comments on the paper A. Charnes, W. W. Cooper, R. J. Niehaus, and D. Sholtz, "Multi-Level Models for Career Management and Resource Planning," in the Proceedings of the NATO Conference "Manpower Planning Models" at Cambridge, England, September 6-10, 1971.

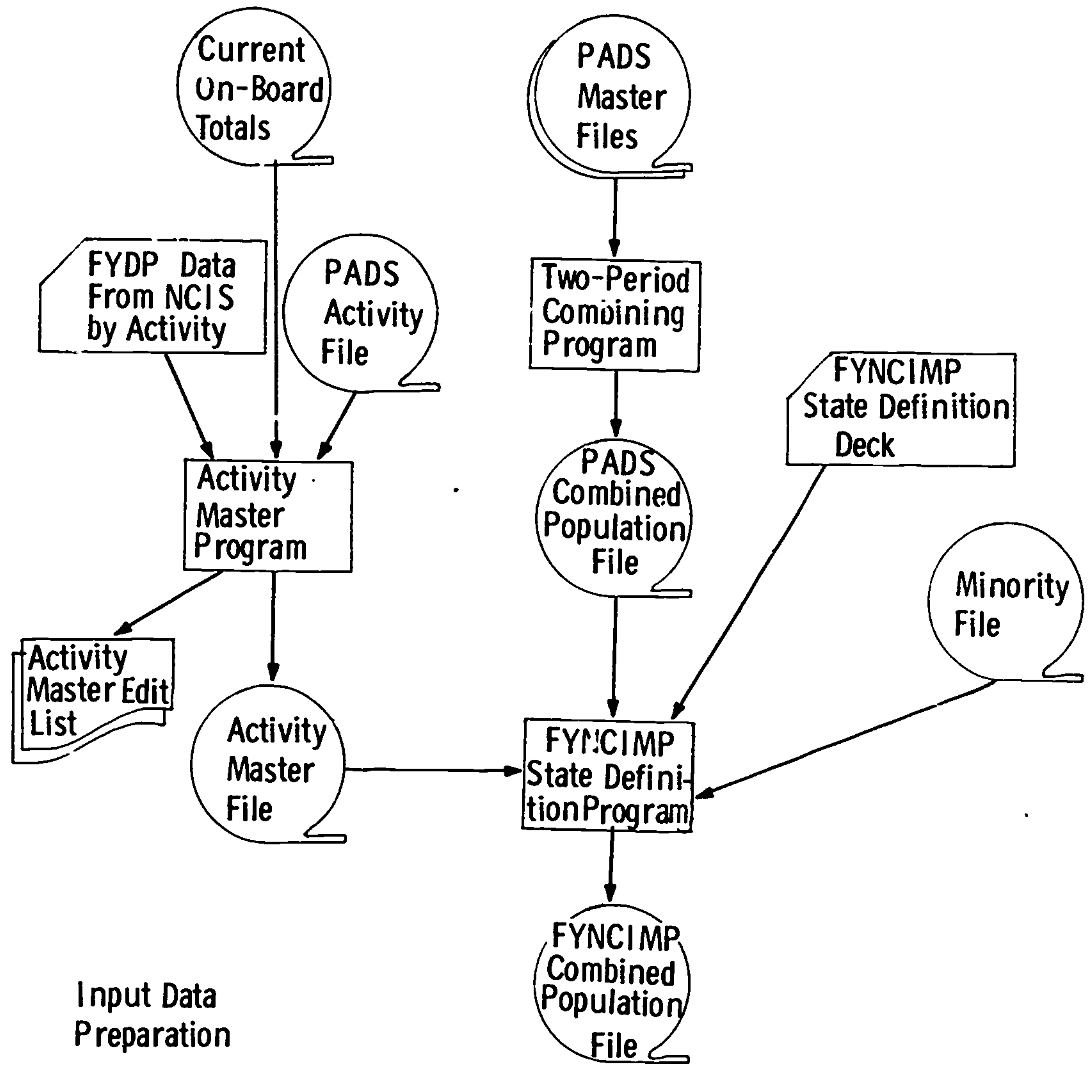


Figure 19. FIVE YEAR NAVY CIVILIAN MANPOWER PLAN (FYNCIMP)

The next section of the FYNCIMP system consists of retirement forecasting. The records of the FYNCIMP combined population file and those on board at the first time period are placed on a separate tape. The FYNCIMP combined population file is also divided into those eligible to retire within the next five years, those that are not, and those on-board at the second time period. The next set of programs calculate attrition due to retirement using the data file of those eligible to retire within the next five years. The outputs of is set of programs consist of a report showing expected retirement rates, numbers expected to retire in each future time period, and a tape containing the number expected to remain in each future time period. A sample of the expected retirement report is shown in Figure 20.

The records of those employees on board at the second or most current time period along with FYDP projections of aggregate civilian manpower requirements are used to obtain a proportioned forecast of gross requirements by state. At this point a report is printed for the manpower planner's use. If he feels that this mix of the work force is not the best, he can make any changes he wishes as long as he stays within the overall total in each future time period. A card-to-tape program then prepares the input used to replace any gross requirements records to be changed. A sample of the gross requirements report is shown as Figure 21.

Additional processing is necessary to obtain the transition rates to be used in the models. Here, the individual employee records of the retirements ineligibles are used in the transition rate program. The output

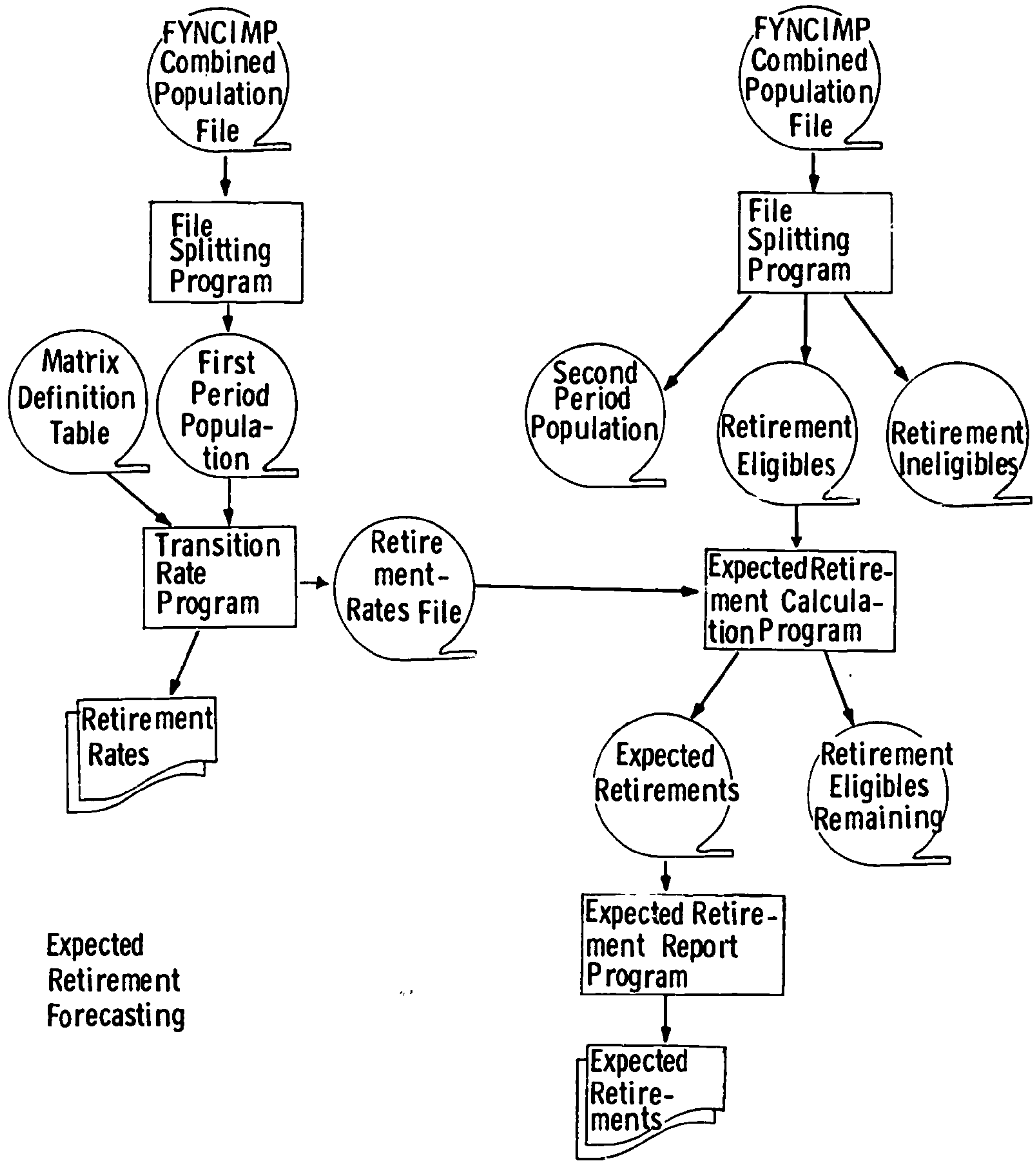
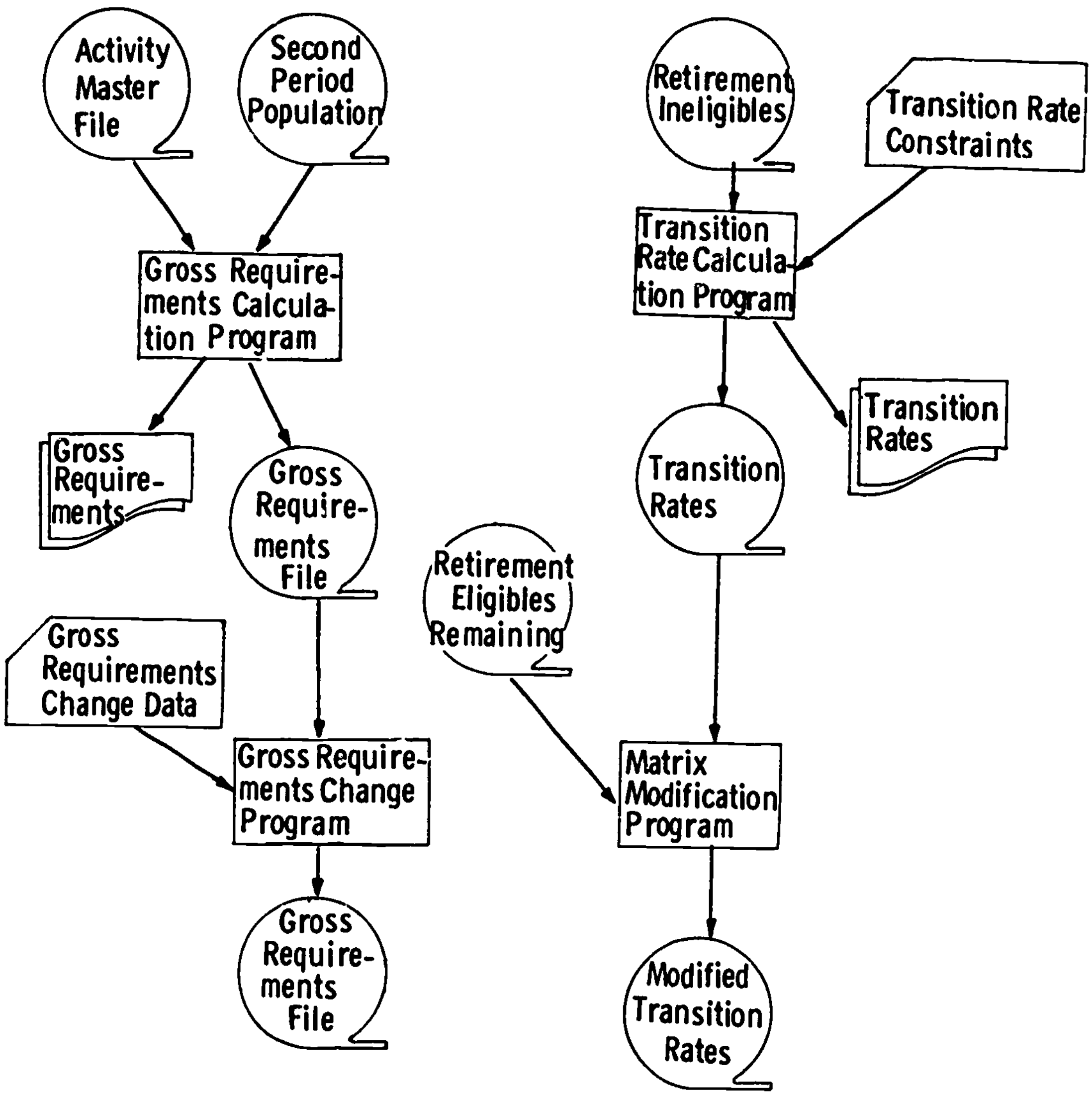


Figure 19—Continued

of this program consists of the transition rates and the number on board in each FYNCIMP state. The transition matrix can also be adjusted by means of a deck of cards to change the relevant rates. The next part of the process consists of adding the number remaining from the retirement eligibles to the transition matrix to obtain a modified transition matrix. The transition rate program can also be used independently as previously discussed.

The final section of the FYNCIMP system is directed toward the generation and solution of the goal programming models. The first program of this section uses a set of control cards plus data tapes containing the transition rates and gross requirements data to generate the input required for linear programming. A tape with the data converted into the standard linear programming format as developed by the LP-90 SHARE users group is obtained. The naming scheme used is given in Figure 22. This format allows the running of the models on most computers capable of using linear programming routines. Through the use of this capability, models have been run on the H-1200, CDC 6600, GE-635, and UNIVAC 1108 computers with little difficulty. Both the career management and multi-level models can be developed using the matrix generator.



Gross Manpower Requirements
and Transition Rates Generation

Figure 19–Continued

The linear programming input data are used in the linear programming routine to obtain the "optimum" results considering the constraints which are present. An example of the type of report obtained from the linear programming software is shown in Figure 23. These are the data from the career management model example shown in the previous chapter. In addition, these results can be produced on a tape, which can be appropriately formatted by a print program. Figure 24 is an example of the formatting of the results shown on Figure 23. In addition to hard-copy output, experimentation is under way with interactive techniques using CRT terminals. This project has just been initiated with the first results expected in the latter part of 1972.

The FYNCIMP system was originally programmed for the Honeywell H-1200 with 65K characters of memory. It is now being converted and revised to a third generation system for use on a UNIVAC 1108. The strategic choice of using COBOL rather than FORTRAN appears to be successful. This has allowed relative ease in terms of the large amounts of input/output processing required. As far as can be determined, this is one of the few large scale applications software systems with substantial amounts of matrix operations

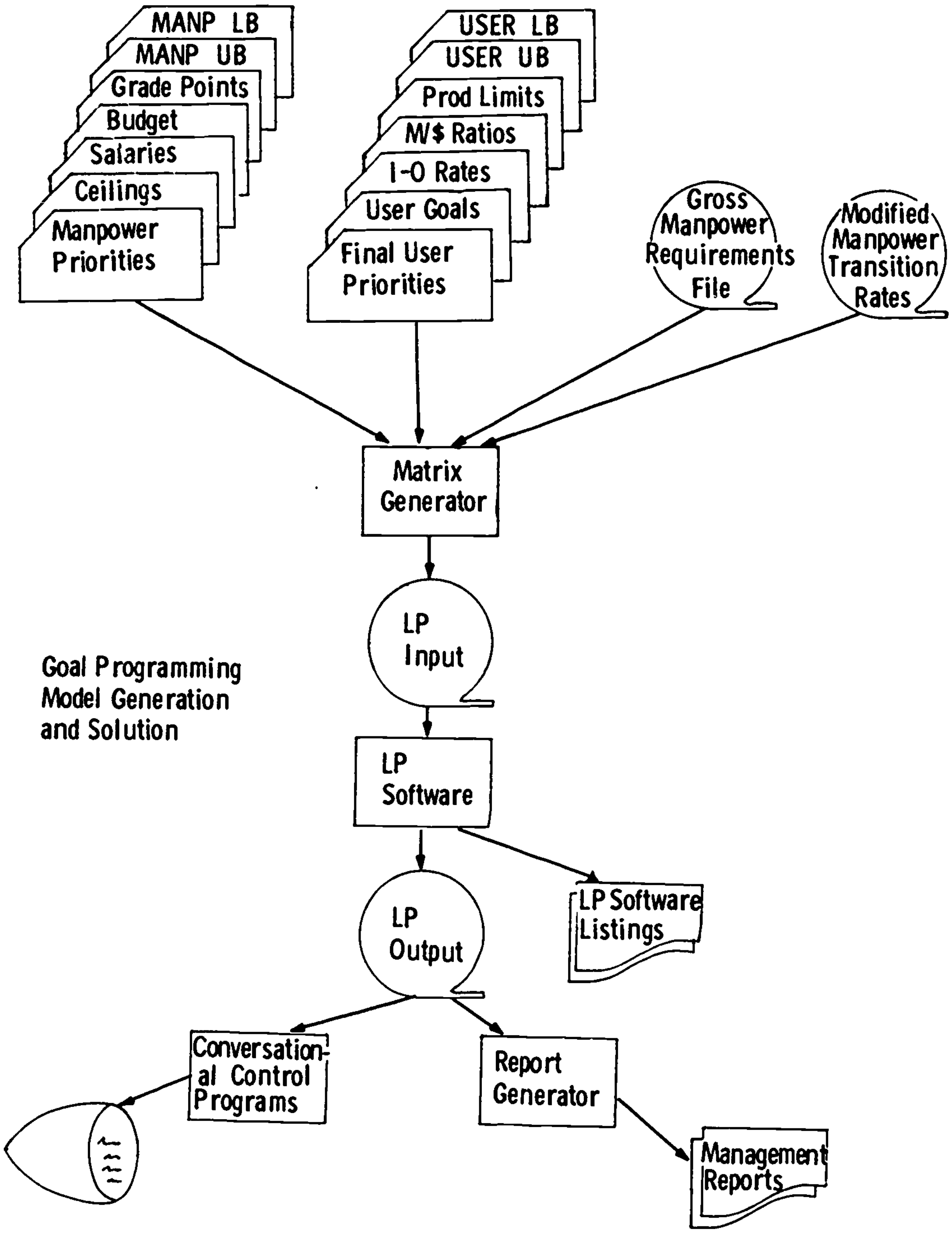


Figure 19–Continued

mixed with high volume input/output processing which has completely and successfully been written in COBOL.

The report formats for the FYNCIMP system were arrived at through the normal interactive process used in computer systems analyses. The first report formats were prepared mainly by the researcher. This was done since the information to be presented was new to both the researcher and the managers. Several comprehensive data studies and systems tests were completed to obtain the reactions of potential users of the system. One of these involved the White Collar population of the naval laboratories. Another centered on the

DEPARTMENT OF THE NAVY
OFFICE OF CIVILIAN MANPOWER MANAGEMENT

DIRECT HIRE U S NATIONAL EXPECTED RETIREMENTS FOR FISCAL YEAR 1973

SHIPYARD

OCCUPATION CLASS	APPRENTS	HELPERS	JOURNYMN	LEADERS	SUPERS	TOTAL
FIRE CONTRCL MCH	0	0	1	0	0	1
ELTRON EQUIP MCH	0	0	2	1	1	4
ELECTRICIAN	0	0	9	1	1	11
PWR PLNT ELTRICN	0	0	3	0	1	4
FABRIC/LETH WRKS	0	0	2	0	1	3
MACHINIST	0	1	7	0	1	9
MACH TOOL OP NEC	0	1	4	0	0	5
GARDNRS + LABRS	0	2	0	0	1	3
MASN PLSTR RCOFR	0	0	2	0	0	2
WELDERS	0	2	17	1	2	22
MOLDER	0	0	1	0	1	2
SHEET MTL WKR	0	0	7	1	1	9
BOILERMAKER	0	0	4	0	1	5
MIL WKRS NEC	0	0	6	1	1	8
MOTION PICT WKRS	0	0	1	0	0	1
PAINTER	0	0	3	0	0	3
PIPE COVERER	0	0	2	0	0	2
PIPE FITTER	0	0	9	1	1	11
PRINTING WORKERS	0	1	1	0	0	2
BOAT REPAIRMAN	0	0	4	0	1	5
MARINE CARPENTER	0	0	4	0	0	4
CARPENTER	0	0	2	0	0	2
WOODWORKER NEC	0	1	0	0	0	1
MILLWRIGHT	0	0	3	0	0	3
FAD EQUIP OPRTRS	0	2	0	0	1	3
PWR PLNT OPRTR	0	0	2	0	1	3
RIGGER	0	1	11	0	2	14
MBL EQP OPTR NEC	0	2	4	0	1	7
SHIP OPRTNG WKRS	0	1	0	0	0	1
MARINE MACHINIST	0	0	4	0	0	4
SHIPFITTER	0	0	9	1	1	11
MAR EQP RPMN NEC	0	0	2	1	1	4
PRODCTION XPDTRS	0	2	0	0	1	3
WAREHOUSE WKRS	0	4	0	0	1	5
TOTALS	0	20	126	8	23	177

Figure 20. FIVE YEAR NAVY CIVILIAN MANPOWER PLAN (FYNCIMP) SYSTEM

DEPARTMENT OF THE NAVY
OFFICE OF CIVILIAN MANPOWER MANAGEMENT

DIRECT HIRE U S NATIONAL GROSS REQUIREMENTS FOR FISCAL YEAR 1974

ALL NAVY

OCCUPATION CLASS	GS 1-4	GS 5-8	GS 9-12	GS 13-15	GS 16-18	TOTAL
ARCHITECT	1	11	175	27	0	214
AEROSPACE ENG	0	91	530	673	13	1,307
CHEMICAL ENG	0	29	177	91	1	298
CIVIL ENG	1	61	613	291	3	969
NAVAL ARCHITECT	0	76	570	521	5	1,172
ELECTRICAL ENG	1	170	750	252	1	1,174
ELECTRONIC ENG	0	360	3,574	2,938	31	6,903
INDUSTRIAL ENG	0	35	307	159	0	501
MECHANICAL ENG	1	418	2,763	1,204	4	4,390
WELDING ENG	0	13	161	113	2	289
NUCLEAR ENG	0	69	354	166	8	597
ENGINEER (NEC)	410	186	1,410	2,648	65	4,719
CHEMIST	0	31	553	310	9	903
METALLURGIST	0	8	125	66	1	200
GEOL + GEOPHYS	0	11	105	29	0	145
OCEANOGRAPHER	0	34	433	145	1	613
CARTOGRAPHER	0	11	205	36	0	252
MATHEMATICIAN	2	158	866	333	3	1,362
PHYSICIST	1	164	1,113	973	37	2,288
PHYS SCIENT NEC	48	27	225	432	49	781
BIOLOGICAL SCIEN	1	11	74	55	2	143
PSYCHOLOGIST	0	17	127	142	1	287
SOCIAL SCIEN NEC	0	10	31	23	1	65
ENG DRAFTSMEN	167	534	15	0	0	716
SURVEYING TECH	13	37	3	0	0	53
ELECTRONICS TECH	32	818	5,253	123	0	6,226
ENGINEERING TECH	408	1,224	6,510	225	0	8,367
IND ENG TECH	14	105	946	16	0	1,081
PHY SCI TECH	221	519	473	16	0	1,229
PROD CONTROLLER	9	790	1,659	122	0	2,580
TECHNICIAN NEC	27	157	407	4	0	595
NURSE	2	952	62	1	0	1,017
OPTOMETRIST	0	0	17	0	0	17
PHARMACIST	0	1	22	0	0	23
PHYSICIAN	0	0	17	68	2	87

Figure 21. FIVE YEAR NAVY CIVILIAN MANPOWER PLAN (FYNCIMP) SYSTEM GROSS REQUIREMENTS REPORT

Experimental Report

The naming scheme for the columns and rows is as follows:

I T U S S L

where

I indicates the specific row or column type

Columns

A	Aboard	Q	Transition rates
B	Hires	R	Ceilings
C	RIFS	S	Salary constraints
D	Manpower positive discrepancies	T	Average grade
E	Manpower negative discrepancies	U	Manpower/Dollar
F	Final user support	V	I/O Rates
G	Producer output	W	User goals
H	User positive discrepancies	X	Producer limits
J	User negative discrepancies		
K	User positive slack		
L	User negative slack	2	Manpower Bounds–lower
M	Right hand side	3	Manpower Bounds–upper
N	Priorities	4	User Bounds–lower
P	Manpower goals	5	User Bounds–upper

T indicates the time period, and may have values from 0 to 9

U indicates the user or producer

SS indicates the job-type

L indicates the level

For the RHS and Objective, L indicates the cardinality of the column or row

Figure 22. MULTI-LEVEL MODEL
LINEAR PROGRAMMING NAMING SCHEME

Blue Collar population of one of the shipyards. The report formats were revised and reprogrammed. After experience had been gained through a number of data studies to meet on-going information requests, the report formats were revised again. The current version of the report formats incorporates the management reaction to over sixty different data studies. These studies range all the way from the Assistant Secretary of the Navy level to the local installation level.

It was found that the chief implementation problem was not the report formats but confidence in the underlying data. The early data studies were technically successful. However, their usefulness to managers was questioned. Later validity studies showed that the transition data were stable allowing projections of on-board manpower to be made with an accuracy of 5 to 10 percent. The importance of all these early studies is that they provided much of the information needed to make the FYNCIMP system workable. An example is the construction of suitable job categories for making the projections. The report formats as mentioned above were also redesigned to condense the length of the reports to a few pages. Many of the ideas learned from these experiments were used in the redesign of the FYNCIMP system for operation on the UNIVAC 1108.

An important factor in the implementation is the conversion of the model mathematics from the net requirements structure to the generalized network structure. This provided much data stability since

PRIMAL OUTPUT

CASE ITERATION 439 OBJECTIVE VALUE -137.74890

LABEL	COST	ACTIVITY	LABEL	COST	ACTIVITY	LABEL	COST	ACTIVITY
E N00001	.000000	-.1377541+03	R30000	.000000	.011172	P50000	.000000	.000008
210012	.000000	.722000	210013	.000000	2.292000	210014	.000000	1.305403
210015	.000000	.000000	210021	.000000	.003000	210022	.000000	.186000
210023	.000000	.454000	210024	.000000	.180214	210025	.000000	.000000
210031	.000000	1.136911	210032	.000000	1.231782	210033	.000000	.099000
210041	.000000	.292000	210042	.000000	.235600	210043	.000000	.019104
220011	.000000	.135462	220012	.000000	.733000	220013	.000000	2.324000
220014	.000000	.840000	220015	.000000	.000000	220021	.000000	.003000
220022	.000000	.194000	220023	.000000	.449000	220024	.000000	.119000
220025	.000000	.000000	220031	.000000	1.798000	220032	.000000	1.033000
220033	.000000	.100544	220041	.000000	.289000	220042	.000000	.157000
220043	.000000	.011000	230011	.000000	.147000	230012	.000000	.740000
230013	.000000	2.345000	230014	.000000	.845461	230015	.000000	.000000
230021	.000000	.003000	230022	.000000	.193000	230023	.000000	.446000
230024	.000000	.117000	230025	.000000	.000000	230031	.000000	1.780000
230032	.000000	1.023000	230033	.000000	.089000	230041	.000000	.296000
230042	.000000	.152832	230043	.000000	.012000	240011	.000000	.144000
240012	.000000	.725000	240013	.000000	2.302000	240014	.000000	.832009
240015	.000000	.000000	240021	.000000	.003000	240022	.000000	.177000
240023	.000000	.431000	240024	.000000	.113000	240025	.000000	.000000
240031	.000000	1.730000	240032	.000000	.994000	240033	.000000	.097000
240041	.000000	.287000	240042	.000000	.155000	240043	.000000	.011000
250011	.000000	.143000	250012	.000000	.719000	250013	.000000	2.291000
250014	.000000	.825000	250015	.000000	.000000	250021	.000000	.003000
250022	.000000	.173000	250023	.000000	.422000	250024	.000000	.111000
250025	.000000	.000000	250031	.000000	1.705000	250032	.000000	.979000
250033	.000000	.085000	250041	.000000	.281000	250042	.000000	.152000
250043	.000000	.011000	310011	.000000	.288000	310012	.000000	.723000
310013	.000000	2.290993	310014	.000000	.353597	310015	.000000	.000000
310021	.000000	.003000	310022	.000000	.186000	310023	.000000	.455000
310024	.000000	.059786	310031	.000000	2.474080	310032	.000000	.843217
310033	.000000	.030000	310041	.000000	.292000	310042	.000000	.090400
310043	.000000	.005896	320011	.000000	.156538	320012	.000000	.733000
320013	.000000	2.324000	320014	.000000	.840000	320015	.000000	.000000
320021	.000000	.003000	320022	.000000	.184000	320023	.000000	.449000
320024	.000000	.119000	320025	.000000	.000000	320031	.000000	1.799000
320032	.000000	1.033000	320033	.000000	.075456	320041	.000000	.299000
320042	.000000	.157000	320043	.000000	.011000	330011	.000000	.147000
330012	.000000	.740000	330013	.000000	2.344000	330014	.000000	.850539
330015	.000000	.000000	330021	.000000	.003000	330022	.000000	.193000
330023	.000000	.446000	330024	.000000	.117000	330025	.000000	.000000
330031	.000000	1.780000	330032	.000000	1.023000	330033	.000000	.099000
330041	.000000	.297000	330042	.000000	.169434	330043	.000000	.012000
340011	.000000	.144000	340012	.000000	.726000	340013	.000000	2.302000
340014	.000000	.831991	340015	.000000	.000000	340021	.000000	.003000

Figure 23. OCMM CAREER MANAGEMENT MODEL LINEAR PROGRAMMING SOLUTION

492-132 O - 73 - 5

LABEL	COST	ACTIVITY	LABEL	COST	ACTIVITY	LABEL	COST	ACTIVITY
340022	.000000	.177000	340023	.000000	.431000	340024	.000000	.114000
340025	.000000	.000000	340031	.000000	1.730000	340032	.000000	.994000
340033	.000000	.097000	340041	.000000	.287000	340042	.000000	.155000
340043	.000000	.011000	350011	.000000	.143000	350012	.000000	.719000
350013	.000000	2.282000	350014	.000000	.825000	350015	.000000	.000000
350021	.000000	.003000	350022	.000000	.173000	350023	.000000	.422000
350024	.000000	.111000	350025	.000000	.000000	350031	.000000	1.704000
350032	.000000	.979000	350033	.000000	.045000	350041	.000000	.291000
350042	.000000	.152000	350043	.000000	.011000	400011	.000000	2.884000
400012	.000000	14.507000	400013	.000000	45.995000	400014	.000000	16.633000
400015	.000000	.304000	400021	.000000	.063000	400022	.000000	3.769000
400023	.000000	9.192000	400024	.000000	2.421000	400025	.000000	.029000
400031	.000000	37.233000	400032	.000000	21.425000	400033	.000000	1.865000
400041	.000000	6.037000	400042	.000000	3.271000	400043	.000000	.239000
410011	.000000	2.729000	410012	.000000	14.450000	410013	.000000	45.819999
410014	.000000	17.047402	410015	.000000	.304000	410021	.000000	.063000
410022	.000000	3.726000	410023	.000000	9.090000	410024	.000000	2.455214
410025	.000000	.028000	410031	.000000	35.435511	410032	.000000	20.942782
410033	.000000	1.805000	410041	.000000	5.846000	410042	.000000	3.245600
410043	.000000	.237104	420011	.000000	2.902462	420012	.000000	14.653000
420013	.000000	46.465000	420014	.000000	16.803000	420015	.000000	.304000
420021	.000000	.062000	420022	.000000	3.685000	420023	.000000	9.989000
420024	.000000	2.372000	420025	.000000	.028000	420031	.000000	35.958000
420032	.000000	20.658000	420033	.000000	1.809544	420041	.000000	5.799000
420042	.000000	3.137000	420043	.000000	.225000	430011	.000000	2.940000
430012	.000000	14.791000	430013	.000000	46.904000	430014	.000000	16.959462
430015	.000000	.304000	430021	.000000	.062000	430022	.000000	3.653000
430023	.000000	8.914000	430024	.000000	2.348000	430025	.000000	.028000
430031	.000000	35.595000	430032	.000000	20.455000	430033	.000000	1.781000
430041	.000000	5.330000	430042	.000000	3.205556	430043	.000000	.234000
440011	.000000	2.896000	440012	.000000	14.514000	440013	.000000	46.032999
440014	.000000	16.648009	440015	.000000	.304000	440021	.000000	.059000
440022	.000000	3.532000	440023	.000000	8.617000	440024	.000000	2.270000
440025	.000000	.028000	440031	.000000	34.557000	440032	.000000	19.881000
440033	.000000	1.731000	440041	.000000	5.732000	440042	.000000	3.107000
440043	.000000	.227000	450011	.000000	2.855000	450012	.000000	14.383000
450013	.000000	45.609999	450014	.000000	16.496000	450015	.000000	.304000
450021	.000000	.058000	450022	.000000	3.460000	450023	.000000	8.443000
450024	.000000	2.224000	450025	.000000	.028000	450031	.000000	34.079000
450032	.000000	19.585000	450033	.000000	1.705000	450041	.000000	5.617000
450042	.000000	3.044000	450043	.000000	.222000	810011	-1.000000	1.277345
810012	-1.000000	2.814843	810013	-1.000000	.725312	810015	-1.000000	.010117
810021	-1.000000	.024132	810022	-1.000000	.467817	810023	-1.000000	.055267
810031	-1.000000	10.654440	810041	-1.000000	1.352204	820011	-1.000000	1.358217
820012	-1.000000	3.598575	820013	-1.000000	3.660707	820014	-1.000000	.089915
820015	-1.000000	.020050	820021	-1.000000	.021580	820022	-1.000000	.591245
820023	-1.000000	.527146	820024	-1.000000	.006654	820025	-1.000000	.000102
820031	-1.000000	10.433944	820032	-1.000000	.835906	820041	-1.000000	1.244365
830011	-1.000000	1.313173	830012	-1.000000	3.563456	830013	-1.000000	3.459232

Figure 23–Continued

LABEL	COST	ACTIVITY	LABEL	COST	ACTIVITY	LABEL	COST	ACTIVITY
83CC14	-1.000000	.463992	83CC15	-1.000000	.020785	83CC21	-1.000000	.009656
83CC22	-1.000000	.590658	83CC23	-1.000000	.541747	83CC24	-1.000000	.071495
83CC25	-1.000000	.000268	83CC31	-1.000000	10.103313	83CC32	-1.000000	.928536
83CC33	-1.000000	.048341	83CC41	-1.000000	1.429513	83CC42	-1.000000	.166292
83CC43	-1.000000	.017811	84CC11	-1.000000	1.245585	84CC12	-1.000000	3.155110
84CC13	-1.000000	2.228208	84CC15	-1.000000	.020129	84CC21	-1.000000	.019320
84CC22	-1.000000	.495446	84CC23	-1.000000	.312021	84CC24	-1.000000	.002260
84CC25	-1.000000	.000316	84CC31	-1.000000	9.355923	84CC32	-1.000000	.444133
84CC33	-1.000000	.018185	84CC41	-1.000000	1.123261	84CC43	-1.000000	.000984
85CC11	-1.000000	1.251913	85CC12	-1.000000	3.293999	85CC13	-1.000000	2.622750
85CC14	-1.000000	.154706	85CC15	-1.000000	.020440	85CC21	-1.000000	.020240
85CC22	-1.000000	.521599	85CC23	-1.000000	.411717	85CC24	-1.000000	.030245
85CC25	-1.000000	.000472	85CC31	-1.000000	9.544217	85CC32	-1.000000	.652718
85CC33	-1.000000	.039603	85CC41	-1.000000	1.161499	85CC42	-1.000000	.033998
85CC43	-1.000000	.002761	C1CC25	-3.000000	.001090	C1CC33	-3.000000	.062745
C1CC42	-3.000000	.002008	C1CC43	-3.000000	.006868	C1CC14	-2.000000	17.047402
D1CC24	-2.000000	.050214	C1CC25	-2.000000	.000000	C1CC32	-2.000000	.153783
D1CC42	-2.000000	.077600	D1CC43	-2.000000	.006104	D2CC33	-2.000000	.010544
D4CC14	-2.000000	.000009	D4CC42	-2.000000	.000000	E1CC11	-2.000000	.144000
E1CC31	-2.000000	.655048	E2CC11	-2.000000	.010538	E3CC14	-2.000000	.002539

END PRIMAL OUTPUT

Figure 23–Continued

DEPARTMENT OF THE NAVY
OFFICE OF CIVILIAN MANPOWER MANAGEMENT

07 JAN 72 PAGE NO 1

OCMM MAJOR OCCUPATION GROUP MODEL

CATEGORY	CODE	ABOARD 1971	1972 ABOARD	1972 HIRES	1972 RIFS	1973 ABOARD	1973 HIRES	1973 RIFS	1974 ABOARD	1974 HIRES	1974 RIFS	1975 ABOARD	1975 HIRES	1975 RIFS	1976 ABOARD	1976 HIRES	1976 RIFS
PROF+TECH G1	0011	2889	2728	1277		2913	1365		2940	1308		2883	1243		2859	1252	
PROF+TECH G2	0012	14507	14245	2609		14653	3722		14791	3561		14514	3195		14383	3284	
PROF+TECH G3	0013	45995	45000			46469			46904	3497		46033	2228		45610	2622	
PROF+TECH G4	0014	16635	17047			16671			16962			16650			16496	152	
PROF+TECH G5	0015	304	304	10		304	20		303	28		304	20		304	20	
ADM+MGR G1	0021	63	63	24		62	21		62	9		59	19		58	20	
ADM+MGR G2	0022	3768	3726	467		3685	591		3653	598		3532	495		3460	521	
ADM+MGR G3	0023	9192	9090	55		8989	529		8914	548		8616	312		8443	411	
ADM+MGR G4	0024	2421	2455			2372	6		2348	78		2270	2		2224	30	
ADM+MGR G5	0025	28	28		1	28			28			28			28		
CLERICAL G1	0031	37283	35819	11037		35958	10568		35595	10102		34597	9355		34079	9544	
CLERICAL G2	0032	21025	20942			20657	812		20455	828		19881	444		19585	692	
CLERICAL G3	0033	1865	1868			1857			1781	6		1731	18		1705	39	
SERVICE G1	0041	6037	5846	1352		5789	1243		5930	1429		5732	1123		5617	1161	
SERVICE G2	0042	3271	3245		2	3137			3206	166		3107			3044	33	
SERVICE G3	0043	239	237		6	229			234	12		227			222	2	
		165917	162643	16831	9	163773	18877		164106	22140		160164	18454		158117	19783	

Figure 24. OCMM CAREER MANAGEMENT MODEL RECRUITING REQUIREMENTS REPORT

gross rather than net numbers are used to make the projections. Equally important is the fact that all the data in the new outputs are in the form familiar to non-mathematically trained users. They do not have to understand the significance of a quantity such as net requirements which were used earlier. The first outputs using the generalized network form of the model have not been circulated widely. All the operational examples have involved a substantial manual effort. It will not be until late 1972 that the model generation process will be mechanized substantially. The examples in the next chapter represent the first published data of the generalized network form of the model. They are being used to acquaint users with the model results prior to the completion of the conversion to the UNIVAC 1108.

The FYNCIMP system design has been discussed with other manpower organizations. This includes representatives from the U. S. Civil Service Commission responsible for the outputs of the Federal Personnel Management Information System (FPMIS). They indicated interest in the possibility of transferral of parts of the FYNCIMP software for Civil Service Commission use. Among the many other organizations which have shown interest include the British Civil Service Department and IBM France.

This chapter discussed the problems of implementing a multi-level model. A description of the first operational prototype of the OCMM Transition Rate subsystem was followed by the steps involved in implementing the complete Five Year Navy Civilian Manpower Plan (FYNCIMP) system. Some of the significant results reported include:

1. the methodology of checkout of large-scale models by "toy" problems and prototypes;

2. the steps undertaken in making the change from a net requirements manpower model to a generalized network manpower model;

3. results from experiments conducted to provide information on the computability of very large model prototypes.

This chapter was concluded with a description of the current version of the FYNCIMP applications software system.

CHAPTER IV
ANALYTICAL DATA STUDIES

An examination of some of the potential capabilities of using the OCMM career management and multi-level models will now be given. This will be done by means of several data studies to illustrate the management capabilities of the models. The first study will consist of a version of the career management model to test various policies concerned with the reduction of average General Schedule grade. This study was prepared for the Director of the Manpower Planning Division of OCMM. The next study will concern the projection of intake requirements for one of the Navy's civilian career programs. The results of this study are being provided by OCMM to the Chief of Naval Material. The final example will be an experimental study which illustrates a state-of-the-art extension of the multi-level model.

The data input for the Major Occupation Group example in Chapter III and the numerical examples given in this chapter were prepared manually. The report formats were tailored to the management requirements of each of the studies. This manual preparation formed the basis for the design specifications of the matrix generator and report writer portions of the FYNCIMP system. With the exception of the extensions to the multi-level model, all of the numerical examples were used to satisfy top level management information requirements. These examples represent a case where research, development, and management practice were served jointly from the same data studies.

The data studies involving the career management model are the first numerical results to be reported of operational uses of the model. These studies were performed for top-level Navy managers familiar with the earlier OCMM modeling applications. These managers have used data produced by the Transition Rate subsystem for several years. They have working experience with the potentials and limitations of the manpower transition data which underlie the career management model. These data studies are helping to ensure an understanding of the models prior to the full implementation of the FYNCIMP system. This is true for the model designers as well as for the model users.

Average Grade Model

In August, 1971, the Office of Management and Budget (OMB) issued Bulletin No. 72-4 establishing a program to reduce the average grade of General Schedule employees of the Federal Government.[1] The purpose of the program was to reduce, Government-wide, the average grade of the General Schedule by one-tenth by the end of Fiscal Year 1972 and by two-tenths by the end of Fiscal Year 1973. In order to achieve the overall objective, the rate of reduction was set according to the amount the average grade had increased between June 30, 1968 and June 30, 1971. For the Navy this meant a reduction of 0.15 in FY 1972 and an additional 0.15 in FY 1973. A strong constraint on the objective was that the average grade reduction had to be accomplished in light of voluntary attrition alone.

The question immediately arose as to the information support required to assist the Navy's managers in responding to this OMB policy. A NACMIS computer program which could be used to provide data on General Schedule distributions was pressed into service. The Transition Rate subsystem of the FYNCIMP system was used to obtain historical data on inter-grade movements for the 1968-1971 period. These data were computed both Navy-wide and by major bureau or command and provided to the pertinent users. Work

[1] *Control of Grade Escalation in the General Schedule*, Office of Management and Budget Bulletin No. 72-4 of August 5, 1971.

was also started on the modification of the career management model to accommodate the average grade constraints. Thus began the first "live" use of the OCMM career management model to test policies which affected the 165,000 General Schedule employees of the Navy.[1]

The career management model was modified as shown in Figure 25. The objective of this model is to minimize discrepancies from the manpower requirements by grade and to favor on-board manpower and new hires over Reductions in Force (RIF's). The average grade constraint is included in the model by requiring that the sum of the number in each grade be multiplied by the total projected population on board. This constraint is made to hold for a given population by setting an additional constraint that the sum of the number in each grade must exactly equal the projected total on-board. Additional constraints of upper and lower bounds on the number in each grade are also set. These ensure against the naive solution of firing large numbers of high grade personnel and replacing them with large numbers of low grade personnel.

In order to use the model, an initial set of goals was developed manually to meet the grade reduction requirements. These were devised considering grade distribution configurations felt to be feasible in terms of past experience. For FY 1972 and FY 1973, these goals are shown in Table 11. These grade goals were initially tested using the following constraints:

1. A total of 160,000 graded employees each year.

2. FY 1972 average grade of 7.6570.

3. FY 1973 average grade of 7.5070.

4. FY 1971 promotion rates were applied for the first six months of FY 1972.

5. FY 1971 promotion rates were applied up to and including GS-10 in both years.

6. No promotions to GS-11 and above for the last half of FY 1972 and all of FY 1973.

7. The number on-board in GS 16-18 to remain constant.

8. A plus or minus deviation of ten people from the goals.

The model was applied with the results as shown in Table 12. These results were used to adjust the grade goals. All of the RIF's were added back to the goals with a corresponding reduction in the number of personnel in the lower grades. The upper and lower bounds were then set equal to the goals for all the grades except for GS 3-8. The model was then re-run allowing it to seek its own level for GS 3-8. This process was repeated several times until the solution in Table 13 was obtained. These results showed, using the constraints previously enumerated, that it was possible to achieve the average grade reduction without RIF's. The grade distribution produced by the model as compared with the initial goals as a percentage of actual 30 June 1971 employment is shown in Table 14.

It was learned after the above results were developed that there was a marked difference in the attrition rates between the first quarter of FY 1971 and the first quarter of FY 1972. This information as shown in Table 15 was obtained through the use of the Transition Rate subsystem. These new rates were used in a one-period model using a manually developed set of goals to meet grade reductions of 0.0 (average grade 7.8071) in FY 1972 with a total strength of 160,000. The first solution as shown in Table 16 was based upon an assumption that the promotion and attrition rates experienced in the September, 1970–September, 1971 period would continue to prevail. In this alternative the model was allowed to deviate only plus or minus ten people from the goal.

[1]This was the first operational use of the OCMM models utilizing the generalized network techniques. All of the earlier studies had involved the "net requirements" version of the models discussed in detail in the previous chapter.

OBJECTIVE: (a) MINIMIZE DISCREPANCIES FROM CIVILIAN MANPOWER REQUIREMENTS BY GRADE AND (b) FAVOR ON-BOARD MANPOWER AND NEW HIRES OVER RIF's.

SUBJ TO:

NUMBER ON BOARD IN EACH GRADE IN EACH PERIOD − POSSIBLE AMOUNT OVER + POSSIBLE AMOUNT UNDER = GRADE REQUIREMENTS

NUMBER ON BOARD IN EACH GRADE AT BASE PERIOD = INITIAL POPULATION

− NUMBER REMAINING IN EACH GRADE FROM PREVIOUS PERIOD + NUMBER ON BOARD IN EACH GRADE AT PRESENT PERIOD − NEW HIRES + RIF's = 0

SUM OF (NUMBER ON BOARD IN EACH GRADE X GRADE) ≤ (AVERAGE GRADE X TOTAL MANPOWER AVAILABLE)

SUM OF (NUMBER ON BOARD IN EACH GRADE) = TOTAL MANPOWER AVAILABLE

Figure 25. NAVY AVERAGE GRADE POLICY TESTING MODEL

TABLE 11

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL

Grade Goals For Initial Run of Average Grade Model

GS	FY 1972	FY 1973
1	523	523
2	3,414	4,214
3	16,736	17,696
4	24,884	25,365
5	19,960	20,440
6	7,741	7,741
7	12,274	12,433
8	2,682	2,682
9	17,413	17,013
10	2,035	1,875
11	18,541	17,821
12	15,746	15,106
13	11,026	10,546
14	4,613	4,293
15	2,094	1,934
16	280	280
17	30	30
18	8	8

TABLE 12

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL
AVERAGE GRADE MODEL

(Constraints as Noted in Text)

FY 1972				FY 1973			
GS	TOTAL ON-BOARD	NEW HIRES	EXCESSES	GS	TOTAL ON-BOARD	NEW HIRES	EXCESSES
1	524	469		1	525	390	
2	3,414	2,389		2	4,214	3,002	
3	16,736	7,218		3	17,696	7,721	
4	24,884	4,145		4	25,365	4,836	
5	19,960	1,752		5	20,440	2,427	
6	7,741	31		6	7,741	249	
7	12,274	1,991		7	12,433	2,466	
8	2,682	134		8	2,682	219	
9	17,413		21	9	17,013		1,133
10	2,035		229	10	1,375		213
11	18,541	14		11	17,821	1,060	
12	15,746		344	12	15,106	667	
13	11,036		367	13	10,546	448	
14	4,612		137	14	4,293	27	
15	2,084		72	15	1,932	21	
16	280	13		16	280	30	
17	30	1		17	30	5	
18	8			18	8		
Average Grade	7.6570				7.5070		

TABLE 13

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL
AVERAGE GRADE MODEL

(Constraints as Noted in Text)

FY 1972				FY 1973			
GS	GOAL	TOTAL ON-BOARD	NEW HIRES	GS	GOAL	TOTAL ON-BOARD	NEW HIRES
1	524	524	470	1	525	525	390
2	3,414	3,414	2,389	2	4,214	4,214	3,002
3	17,863	17,683	8,345	3	18,796	18,796	8,216
4	24,685	24,685	4,091	4	24,992	25,455	4,746
5	19,762	19,762	1,544	5	20,067	20,067	2,186
6	7,741	7,741	31	6	7,741	7,741	265
7	11,019	11,019	591	7	10,855	10,392	1,332
8	2,548	2,548		8	2,414	2,414	110
9	17,434	17,434		9	17,920	17,920	273
10	2,035	2,035		10	2,315	2,315	
11	18,541	18,541		11	17,821	17,821	1,060
12	15,808	15,808		12	15,106	15,106	610
13	11,403	11,403		13	10,546	10,546	112
14	4,749	4,749		14	4,393	4,393	
15	2,156	2,156		15	1,977	1,977	
16	280	280	13	16	280	280	30
17	30	30	1	17	30	30	5
18	8	8		18	8	8	
Average Grade	7.6570				7.5070		

Using the same goals again, the model was re-run suppressing promotions to GS-11 and above for the last half of FY 1972. These results are shown in Table 17. These goals were adjusted to eliminate RIF's since a major objective of the study was to find a grade distribution which attained a reduction in both average grade and total employment without RIF's. This new configuration was tested using an average grade constraint of 7.8071. However, it was not possible to obtain a feasible solution within that average grade constraint.

The model was re-run again. This time it was allowed to seek its own average grade level with an upper limit of 7.9000. In this alternative, the lower and upper bounds were set at a deviation of plus or minus ten people from the goals in all grades except for the following:

1. GS-1 -0 —lower boundary
   +50 —upper boundary

2. GS-2–5 ± 5% deviation on lower and upper boundaries

3. GS-16–18 lower boundary = upper boundary = goal.

The revised goals and resulting solution under these conditions is shown in Table 18. The average grade of this configuration if 7.8873. In order to determine if this average grade could be further reduced, a subsequent series of tests were run using average grade constraints of 7.825, 7,850, 7.860 and 7.870. None of these

TABLE 14

NAVY AVERAGE GRADE MODEL
COMPARISON OF FINAL MODEL DATA AND INITIAL GOALS
AS A PERCENT OF 30 JUNE 1971 EMPLOYMENT

(June 1970–June 1971 Transition Data)

GRADE	June 30, 1971 Actual	FY 1972		FY 1973	
		Initial Goals %	Model Output %	Initial Goals %	Model Output %
1	211	247.9	248.3	247.9	248.8
2	3,036	112.5	112.5	138.8	138.8
3	16,154	103.6	110.6	109.5	116.4
4	25,377	98.1	97.8	100.0	100.3
5	20,151	99.1	98.1	101.4	99.6
6	8,007	96.7	96.7	96.7	96.7
7	12,696	96.7	85.6	97.9	81.9
8	2,774	96.7	91.6	96.7	87.0
9	18,237	95.0	95.1	92.8	97.8
10	2,188	93.0	93.0	85.7	105.8
11	19,974	92.8	92.8	89.2	89.2
12	17,032	92.4	92.8	88.7	88.7
13	12,018	91.8	94.9	87.8	87.8
14	4,971	92.8	95.5	86.4	88.4
15	2,266	92.4	95.1	85.3	87.[illegible]
16	289	96.9	96.9	96.9	96.9
17	31	96.8	96.8	96.8	96.8
18	8	100.0	100.0	100.0	100.0

TABLE 15

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL
ANNUAL ATTRITION RATES

GS	30 June 1970–30 June 1971 Percent of 30 June 1970 Population	30 September 1970–30 September 1971 Percent of 30 September 1970 Population
1	39.5	35.6
2	36.4	38.6
3	25.2	22.6
4	17.6	16.4
5	14.2	11.2
6	11.0	8.6
7	12.7	10.5
8	9.8	6.7
9	10.7	5.6
10	8.4	6.4
11	9.6	5.3
12	8.4	4.0
13	8.5	4.6
14	7.4	3.2
15	8.3	4.0
16	10.7	6.6
17	16.0	11.1
18	0.0	0.0

TABLE 16

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL
AVERAGE GRADE MODEL

(30 September 1970–30 September 1971 Transition Rates, Other Constraints as Noted in Text)
FY 1972

GS	GOALS	PROJECTED ON-BOARD	NEW HIRES	EXCESSES
1	203	203	143	
2	2,934	2,934	1,997	
3	15,616	15,616	5,789	
4	24,532	24,532	3,430	
5	19,480	19,480	534	215
6	7,741	7,751		
7	12,274	12,274	1,662	
8	2,682	2,682	19	
9	17,717	17,707	955	36
10	2,115	2,115		523
11	19,309	19,309		853
12	16,466	16,466		695
13	11,618	11,618		319
14	4,805	4,795		136
15	2,190	2,200		4
16	280	280		2
17	30	30		2
18	8	8		

TABLE 17

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL
AVERAGE GRADE MODEL

(30 September 1970–30 September 1971 Transition Data, Other Constraints as Discussed in Text)
FY 1972

GS	GOALS	PROJECTED ON-BOARD	NEW HIRES	EXCESSES
1	203	203	143	
2	2,934	2,934	1,997	
3	15,616	15,616	5,789	
4	24,532	24,532	3,429	
5	19,480	19,480	534	
6	7,741	7,751		215
7	12,274	12,264	1,652	
8	2,682	2,672	9	
9	17,717	17,717		446
10	2,115	2,115		233
11	19,309	19,309		64
12	16,466	16,476		349
13	11,618	11,628		261
14	4,805	4,795		168
15	2,190	2,180		76
16	280	280	3	
17	30	30		
18	8	8		

TABLE 18

NAVY GENERAL SCHEDULE CIVILIAN PERSONNEL
AVERAGE GRADE MODEL

(30 September 1970–30 September 1971 Transition Data, Other Constraints as Noted in Text)
FY 1972

GS	GOALS	PROJECTED ON-BOARD	NEW HIRES	EXCESSES
1	203	203	143	NONE
2	2,653	2,653	1,716	
3	14,799	14,799	4,972	
4	24,050	24,050	2,947	
5	19,480	19,480	534	
6	7,966	7,966		
7	12,032	12,032	1,420	
8	2,672	2,672	9	
9	18,163	18,163		
10	2,348	2,348		
11	19,383	19,383		
12	16,825	16,825		
13	11,889	11,889		
14	4,963	4,963		
15	2,256	2,256		
16	280	280	3	
17	30	30		
18	8	8		

were able to satisfy both the constraints of lowering the graded civilian employment level to 160,000 and a condition of no RIF's.

Career Management Model

The next example concerns the civilian intake requirements for the Navy's procurement career program.[1] The purpose of this example is to illustrate a use of the career management model with smaller size manpower categories. Experience has shown that the manpower categories should in most cases contain at least fifty or more people. The transition data are shown in Table 19. These data were computed by first using the occupation code change capability of the FYNCIMP system. This capability is used by reading into the computer a dictionary of the desired codes as they relate to the Civil Service Commission occupation codes. Provision has been made for any combination of occupational and grade/level grouping aggregations as long as there are 100 or fewer occupation groups and 5 or fewer grade/level groupings. In this case the standard FYNCIMP grade groupings (i GS 1-4, 5-8, 9-12, 13-15, 16-18) were used with the following codes:

CSC Occupation Code	Career Management Code
GS-1101	448
GS-1102	449
GS-1103	450
GS-1150	451

[1]For a comprehensive study of the pr... ..s of l... stics manpower management, see K. C. Taylor, "Aggregate Manpower Management of the Logistics Work Force," *The Journal of Navy Civilian Manpower Management*, Vol. III, No. 3, Fall, 1969.

TABLE 19

RELATIVE FREQUENCY OF MOVEMENT INTER-STATE
NAVY CIVILIAN PROCUREMENT MANAGEMENT CAREER PROGRAM
FROM JUNE 1970 TO JUNE 1971

SEX BOTH — BIRTHDATE 000 TO 000 — SERVICE COMPUTATION DATE 000 TO 000 — MAJOR CLAIMANT — ACTIVITY — GRADE LEVEL

OCCUPATION	NO AT JUN70	44820	44830	44840	44850	44920	44930	44940	44950	45020	45030	45040	45050	45120	45130	45140	45150	EXITS
44820	73	.761	.110				.014											.095
44830	224		.821	.009			.009				.009							.152
44840	83			.843														.157
44850	2				.500													.500
44920	325					.618	.212											.170
44930	1423		.001				.842	.021										.136
44940	626			.002			.003	.871	.005									.120
44950	8								.875									.125
45020	15									.667	.067							.266
45030	95										.800				.011			.189
45040	0																	
45050	0																	
45120	5													.600	.200			.200
45130	533		.006	.002			.002				.006			.002	.782	.008		.193
45140	218							.005								.872		.124
45150	0																	
I GAINS		12	21	3	0	54	34	0	0	2	10	0	0	2	27	4	0	
E GAINS		4	29	9	0	33	49	10	0	0	4	0	0	4	19	2	0	
NO AT JUN71		75	247	86	1	288	1356	586	10	12	96	0	0	10	465	200	0	

TOTAL AT JUN 70 IS 3,630 TOTAL AT JUN 71 IS 3,432

Forecasted manpower goals are shown in Table 20. This forecast was made using proportioned estimates from a Navy-wide forecast by major occupation group. Thus, the data for the procurement career program are consistent with the Navy civilian strength projections included in the Five Year Defense Program.

TABLE 20

NAVY CIVILIAN PROCUREMENT MANAGEMENT
CAREER PROGRAM

	On-Board			Projected Requirements			
	Job Category	30 June 1971	30 June 1972	30 June 1973	30 June 1974	30 June 1975	30 June 1976
GS-1101	5-8	75	74	76	76	75	74
General Business	9-12	247	244	252	251	247	245
	13-15	86	85	88	88	86	85
GS-1102	5-8	288	286	295	294	289	286
Contract &	9-12	1,356	1,343	1,387	1,382	1,357	1,345
Procurement	13-15	586	581	600	598	587	581
GS-1103	5-8	12	12	12	12	12	12
Industrial Property Management	9-12	96	95	99	98	96	96
GS-1150	5-8	10	10	10	10	10	10
Industrial	9-12	465	461	476	474	465	461
Specialist	13-15	200	198	205	204	200	198

The results of the model solution are given in Table 21. These data are being used as preliminary projections of trainee intake requirements by the administrator of the career program. These are the first results of the use of the generalized network form of the model to support career planning. Application of the model is being extended to other career programs.

Multi-Level Model With Final User and Manpower Goals

The remainder of this chapter concerns an example of the multi-level model. This example involves a model structure with goal programming features for both final user and manpower requirements. In addition to this new structural change, this test of the model incorporates actual Navy data where possible. Thus, this numerical example represents the next step from the toy problem presented at the NATO Conference at Cambridge. It is an improvement of the state-of-the-art in terms of: (a) model structural development, and (b) testing of computability with data which approximate that presently available.

The first step in moving the toy problem towards implementation is to use live data sources to approximate all the parameters and coefficients used in the model. The purpose of such a step is to prove the computability of the model with live data. For this reason old data were used which have been overtaken by events. Also, in the case of projected data, rather bland assumptions were made which have little connection with actual forecasts. Some of the historical data have been adjusted and the names of the various users and producers of the input-output submodel have been replaced by codes. The resource relationships are stated in terms of "resource units" rather than dollars. Thus, all these steps have been taken to assure that projected Navy data would not be compromised. The point to be remembered is that all of the data used in this example could be obtained from existing, often-used data files. This is important since the crux of implementing most operations research techniques rests on the obtainability and reliability of the model coefficients.

TABLE 21

DEPARTMENT OF THE NAVY
OFFICE OF CIVILIAN MANPOWER MANAGEMENT

24 FEB 72

PROCUREMENT MANAGEMENT CAREER PROGRAM RECRUITING REQUIREMENTS

CATEGORY	CODE	ABOARD 1971	1972 ABOARD	1972 HIRES	1972 RIFS	1973 ABOARD	1973 HIRES	1973 RIFS	1974 ABOARD	1974 HIRES	1974 RIFS	1975 ABOARD	1975 HIRES	1975 RIFS	1976 ABOARD	1976 HIRES	1976 RIFS
GS1101 5-8	0012	75	74	13		76	15		75	16		75	15		74	15	
GS1101 9-12	0013	247	244	17		252	28		251	31		247	28		245	29	
GS1101 13-15	0014	86	85	1		88	5		88	9		86	7		85	8	
GS1102 5-8	0022	288	286	97		295	107		294	111		289	107		286	107	
GS1102 9-12	0023	1356	[illegible]	43		1387	100		1382	145		1357	124		1345	135	
GS1102 13-15	0024	585	[illegible]	4		600	28		598	45		587	36		581	40	
GS1103 5-8	0032	12	12	3		10	1		6			4			12	9	
GS1103 9-12	0033	96	95	3		99	8		98	3		96	2		96	4	
GS1150 5-8	0042	10	10	2		10	7		6			5			8	3	
GS1150 9-12	0043	465	461	50		476	59		472	97		465	93		461	95	
GS1150 13-15	0044	200	198	6		205	15		204	21		200	18		198	19	
		3421	3399	239		3498	379		3475	478		3411	430		3391	464	

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Figure 26 shows the new multi-level model structure which includes the additional manpower goal conditions. Another set of priority weights has been included. There are priority weights on final user goal discrepancies, new hires, RIF's, and on manpower goal discrepancies. The model can now be used simultaneously to consider each of these sets of variables relative to the value of the corresponding priority weight. In the example which follows, the relative priorities are set such that: (a) the meeting of the final user goals is the most important, (b) it is more desirable not to RIF personnel than to meet the manpower vacancies, and (c) on-board personnel will be considered before any hiring takes place.

The resource related historical data used in the example were obtained from early test runs of the Chief of Naval Operations NARM model.[1] The manpower related data were developed using the Transition Rate subsystem of the Five Year Navy Civilian Manpower Plan (FYNCIMP) system. All of these data should be regarded as experimental.

The resource related data are shown in Table 22. Only one final user was included in order to keep the example to a reasonable size. Table 22 also includes the final user-producer input-output rates and the manpower/resource unit rates. The support-on-support rates are given in Table 23.

The manpower transition data are given in Table 24. These are actual June, 1970–June, 1971 data for the major occupation groups. In succeeding prototype tests these occupation groups will be expanded to eight categories. Feedback from potential users indicates that it would be better tc break the Professional and Technical group into Scientists and Engineers, Sub-Professional Technicians, and Other Professionals. The Blue Collar category will also be expanded to craftsman and Operatives, and to Gardeners and Laborers.

The remaining starting conditions for the numerical example were all hypothetical. This included setting the Final User requirements to a constant 21,000 resource units and the manpower ceilings to a constant 295,000 billets. The hypothetical resource constraints on the Producers is given in Table 25 and the hypothetical manpower requirements are given in Table 26.

The data were then put into the linear programming matrix shown in Figure 27. In order to incorporate the weighting scheme discussed earlier, the following relative priorities were used:

Variable	Relative Weight
Final User Goal Discrepancy	100
RIF's	3
Manpower Goal Discrepancy	2
New Hires	1
On-Board Manpower	0

The solution to this model is given in Tables 27 through 30. If desired, additional alternatives could now be run to test various management policies. The importance of these results is that they show that it is computationally possible with existing data sources to use this extended form of the multi-level model. In all, five sub-models were solved as well as the overall system solution.

The three numerical examples given in this chapter conclude the technical portion of this report. These examples provide the first published results of (a) initial operational experience with the OCMM career management model and (b) methods employed to develop a new extension of the multi-level model. The next and final chapter will provide a summary of conclusions of the research.

[1]This Navy Requirements (NARM) model was developed by the Center for Naval Analyses for CNO as explained in Chapter II.

OBJECTIVE: MINIMIZE COST (MEASURED BY RELATIVE PRIORITIES) OF BEING OVER/UNDER FINAL USER SUPPORT REQUIREMENTS AND OF RECRUITING/REDUCTIONS OF CIVILIAN MANPOWER AND COST (MEASURED BY SECOND SET OF RELATIVE PRIORITIES) OF BEING OVER/UNDER MANPOWER REQUIREMENTS.

SUBJECT TO CONSTRAINTS OF:

TOTAL AMOUNT EACH FINAL USER SUPPORTED - POSSIBLE AMOUNT OVER + POSSIBLE AMOUNT UNDER = EACH FINAL USER SUPPORT REQUIREMENT

- TOTAL AMOUNT EACH FINAL USER SUPPORTED + SUM OF (PROPORTION OF OUTPUT EACH PRODUCER PROVIDES TO EACH FINAL USER X TOTAL OUTPUT OF EACH PRODUCER) = 0

TOTAL OUTPUT OF EACH PRODUCER ≤ BUDGET OF EACH PRODUCER

SUM OF (CIVILIAN MANPOWER REQUIRED FOR EACH UNIT OF SUPPORT X TOTAL OUTPUT OF EACH PRODUCER) - REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER = 0

NUMBER ON-BOARD IN EACH JOB CATEGORY IN EACH PERIOD - POSSIBLE AMOUNT OVER + POSSIBLE AMOUNT UNDER = MANPOWER REQUIREMENTS BY JOB CATEGORY

REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER TOTAL CIVILIAN MANPOWER AVAILABLE

CIVILIAN MANPOWER ON-BOARD AT START = INITIAL POPULATION

- REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER FROM PREVIOUS PERIODS + CIVILIAN MANPOWER ON-BOARD AT PRESENT PERIOD - REQUIRED CIVILIAN MANPOWER PROVIDED BY NEW HIRES + EXCESS CIVILIAN MANPOWER = 0

Figure 26. DYNAMIC MULTI-LEVEL MODEL STRUCTURE WITH BOTH FINAL USER AND MANPOWER GOALS

TABLE 22

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
HISTORICAL RESOURCE USAGE DATA
(In Resource Units)

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4	FINAL USER	TOTALS	FINAL USER/PRODUCER I-O RATE
PRODUCER 1	87.87	117.89	71.85	203.88	512.54	993.94	0.515574
PRODUCER 2	39.95	92.84	32.29	7.75	302.10	474.93	0.630941
PRODUCER 3	99.01	158.00	74.62	254.19	1380.03	1965.85	0.702017
PRODUCER 4	47.92	25.05	39.56	196.13	210.35	518.98	0.405314
CIVILIAN MANPOWER	57,235	11,306	23,952	206,881			
MANPOWER/ RESOURCE UNIT	57.5839	23.6562	12.8404	398.6300			

TABLE 23

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
HISTORICAL SUPPORT-ON-SUPPORT RATES

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PRODUCER 1	0.088405	0.118609	0.072288	0.205123
PRODUCER 2	0.084118	0.332681	0.067989	0.016382
PRODUCER 3	0.053650	0.080372	0.037958	0.129303
PRODUCER 4	0.092335	0.048268	0.076226	0.377914

TABLE 24

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
MANPOWER TRANSITION RATES

JUNE 1970–JUNE 1971

	PROF & TECH	MGR & ADM	CLERICAL	SERVICE	BLUE COLLAR	EXIST
Prof & Tech	.897	.004	.006			.093
Mgr & Adm	.016	.910	.018			.056
Clerical	.020	.009	.785	.001		.184
Service	.002		.007	.842		.149
Blue Collar					.845	.155

TABLE 25

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
HYPOTHETICAL PRODUCER RESOURCE LIMITS
(In Resource Units)

	BASE PERIOD	PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 4	PERIOD 5
PRODUCER 1	993.94	1000.00	1000.00	1000.00	1000.00	1000.00
PRODUCER 2	474.93	500.00	500.00	500.00	500.00	500.00
PRODUCER 3	1965.85	1900.00	1900.00	1900.00	1900.00	1900.00
PRODUCER 4	518.98	500.00	500.00	500.00	500.00	500.00

TABLE 26

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
HYPOTHETICAL MANPOWER REQUIREMENTS DATA

	BASE PERIOD	PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 4	PERIOD 5
PRODUCER 1						
Prof & Tech	2,550	2,600	2,650	2,700	2,750	2,800
Mgr & Adm	1,185	1,200	1,250	1,300	1,350	1,400
Clerical	14,000	14,000	14,000	14,000	14,000	14,000
Service	5,000	5,000	5,000	5,000	5,000	5,000
Blue Collar	35,000	34,000	33,000	32,000	31,000	30,000
PRODUCER 2						
Prof & Tech	1,000	1,100	1,150	1,200	1,250	1,300
Mgr & Adm	400	500	550	600	650	700
Clerical	5,600	5,500	5,400	5,300	5,200	5,100
Service	1,000	1,000	1,100	1,200	1,500	1,500
Blue Collar	3,306	3,000	3,000	3,000	3,000	3,000
PRODUCER 3						
Prof & Tech	8,452	8,000	7,500	7,500	7,500	7,500
Mgr & Adm	1,000	1,000	1,000	1,000	1,000	1,000
Clerical	8,000	8,000	7,500	7,500	7,500	7,500
Service	500	500	500	500	500	500
Blue Collar	6,000	6,000	6,000	6,000	6,000	6,000
PRODUCER 4						
Prof & Tech	52,000	50,000	50,000	50,000	50,000	50,000
Mgr & Adm	10,000	10,000	10,000	10,000	10,000	10,000
Clerical	25,000	25,000	25,000	25,000	25,000	25,000
Service	3,000	2,500	2,500	2,500	2,500	2,500
Blue Collar	126,881	120,000	120,000	120,000	120,000	120,000
TOTALS	309,874	298,900	297,100	296,300	295,700	294,800

TABLE 27

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
FINAL USER SOLUTION DATA
(In Resources Units)

		REQUIRED	ACTUAL	DIFFERENCE
FINAL USER 1	PERIOD 1	2,100	2,100	
FINAL USER 1	PERIOD 2	2,100	2,100	
FINAL USER 1	PERIOD 3	2,100	2,100	
FINAL USER 1	PERIOD 4	2,100	2,100	
FINAL USER 1	PERIOD 5	2,100	2,100	

	Positive Final User Goal Discrepancy	Negative Final User Goal Discrepancy	Final User Support	Producer Output	On-Board Manpower	New Hire Manpower	Excess Manpower	Positive Manpower Goal Discrepancy	Negative Manpower Goal Discrepancy	Sign	RHS
Relative Priorities	α	α				β	γ	δ	δ		
Final User Goals	-1 -1	1 1	1 1							= =	Final User Support Requirements
Support Usage Rates			-1 -1	I-O Rates I-O Rates						= =	O O
Production Limits				1 1						≤ ≤	Producer Budgetary Limits
Manpower/ Producer Output				$(m/\$)_1$ $(m/\$)_2$	-1 -1					= =	O O
Manpower Goals					1 1			-1 -1	1 1	= =	Manpower Requirements
Total Manpower Constraints					1111 1111					≤ ≤	Total Manpower Available
Manpower Attrition					1 -M 1 -M 1	 -1 -1	 1 1			= = =	Init. Pop O O

Figure 27. DYNAMIC MULTI-LEVEL MODEL WITH BOTH FINAL USER AND MANPOWER GOALS LINEAR PROGRAMMING MATRIX

TABLE 28

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
PRODUCER BUDGETARY RESOURCES SOLUTION DATA
(In Resource Units)

	AVAILABLE	USED	UNUSED
PERIOD 1			
PRODUCER 1	1,000	987	13
PRODUCER 2	500	410	90
PRODUCER 3	1,900	1,610	290
PRODUCER 4	500	500	
PERIOD 2			
PRODUCER 1	1,000	971	29
PRODUCER 2	500	473	27
PRODUCER 3	1,900	1,564	336
PRODUCER 4	500	500	
PERIOD 3			
PRODUCER 1	1,000	955	45
PRODUCER 2	500	478	22
PRODUCER 3	1,900	1,572	328
PRODUCER 4	500	500	
PERIOD 4			
PRODUCER 1	1,000	940	60
PRODUCER 2	500	490	10
PRODUCER 3	1,900	1,572	328
PRODUCER 4	500	500	
PERIOD 5			
PRODUCER 1	1,000	924	76
PRODUCER 2	500	490	10
PRODUCER 3	1,900	1,583	317
PRODUCER 4	500	500	

TABLE 29

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
MANPOWER SOLUTION DATA

		ABOARD	HIRES	EXCESS	GOAL	DISCREPANCY
Period 1						
PRODUCER 1	P&T	2,600	4		2,600	
	M&A	1,215			1,200	+15
	CL	14,000	2,924		14,000	
	SE	5,000	776		5,000	
	BC	34,000	4,425		34,000	
PRODUCER 2	P&T	1,100	83		1,100	
	M&A	500	82		500	
	CL	4,422			5,500	-1078
	SE	883			1,000	-117
	BC	2,794	36		3,000	-206
PRODUCER 3	P&T	7,758			8,000	-242
	M&A	1,000			1,000	
	CL	6,360		16	8,000	-1640
	SE	421			500	-79
	BC	5,070			6,000	-930
PRODUCER 4	P&T	50,000	2,690		50,000	
	M&A	9,533			10,000	-467
	CL	20,151			25,000	-4849
	SE	2,551			2,500	+51
	BC	117,080	9,866		120,000	-2920
Period 2						
PRODUCER 1	P&T	2,642			2,650	-8
	M&A	1,250	8		1,250	
	CL	14,000	2,923		14,000	
	SE	5,000	776		5,000	
	BC	33,000	4,270		33,000	
PRODUCER 2	P&T	1,150	65		1,150	
	M&A	550	51		550	
	CL	5,400	1,903		5,400	
	SE	1,100	352		1,100	
	BC	3,000	639		3,000	
PRODUCER 3	P&T	7,500	397		7,500	
	M&A	1,000	2		1,000	
	CL	5,067			7,500	-2433
	SE	500	139		500	
	BC	5,963	1,679		6,000	-37

TABLE 29–Continued

		ABOARD	HIRES	EXCESS	GOAL	DISCREPANCY
Period 2–Continued						
PRODUCER 4	?&T	50,000	4,589		50,000	
	M&A	10,608		81	10,000	+608
	CL	16,328			25,000	-8672
	SE	2,500	331		2,500	
	BC	119,879	20,947		120,000	-121
Period 3						
PRODUCER 1	P&T	2,680			2,700	-20
	M&A	1,300	26		1,300	
	CL	14,000	2,923		14,000	
	SE	5,000	776		5,000	
	BC	32,000	4,115		32,000	
PRODUCER 2	P&T	1,200	49		1,200	
	M&A	600	46		600	
	CL	5,300	1,031		5,300	
	SE	1,200	268		1,200	
	BC	3,000	465		3,000	
PRODUCER 3	P&T	7,500	654		7,400	
	M&A	1,000	14		1,000	
	CL	5,130	1,081		7,500	-2370
	SE	500	74		500	
	BC	6,000	96		6,000	
PRODUCER 4	P&T	50,000	4,649		50,000	
	M&A	10,000			10,000	
	CL	16,815	3,473		5,000	-8185
	SE	2,500	379		2,500	
	BC	120,000	18,702		120,000	
Period 4						
PRODUCER 1	P&T	2,774		30	2,750	+24
	M&A	1,350	30		1,350	
	CL	14,000	2,922		14,000	
	SE	5,000	776		5,000	
	BC	31,000	3,960		31,000	
PRODUCER 2	P&T	1,250	56		1,250	
	M&A	650	52		650	
	CL	5,200	1,008		5,200	
	SE	1,500	484		1,500	
	BC	3,000	465		3,000	

TABLE 29–Continued

		ABOARD	HIRES	EXCESS	GOAL	DISCREPANCY
Period 4—Continued						
PRODUCER 3	P&T	7,500	664		7,500	
	M&A	1,000	14		1,000	
	CL	5,124	1,025		7,500	-2376
	SE	500	74		500	
	BC	6,000	930		6,000	
PRODUCER 4	P&T	50,000	4,649		50,000	
	M&A	10,000	549		10,000	
	CL	16,815	3,101		25,000	-8185
	SE	2,500	378		2,500	
	BC	120,000	18,600		120,000	
Period 5						
PRODUCER 1	P&T	2,800			2,800	
	M&A	1,400	34		1,400	
	CL	14,000	2,920		14,000	
	SE	5,000	776		5,000	
	BC	30,000	3,805		30,000	
PRODUCER 2	P&T	1,300	61		1,300	
	M&A	700	57		700	
	CL	5,100	983		5,100	
	SE	1,500	231		1,500	
	BC	3,000	465		3,000	
PRODUCER 3	P&T	6,847			7,500	
	M&A	1,000	14		1,000	
	CL	6,002	1,908		7,500	-1498
	SE	426			500	-73
	BC	6,000	930		6,000	
PRODUCER 4	P&T	45,352			50,000	-4649
	M&A	10,000	549		10,000	
	CL	25,000	11,286		25,000	
	SE	2,500	378		2,500	
	BC	116,464	15,064		120,000	-3536

TABLE 30

MULTI-LEVEL MODEL TEST USING CNA AND OCMM DATA
SUPPORT-ON-SUPPORT DATA

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PERIOD 1				
PRODUCER 1	87	117	71	202
PRODUCER 2	34	136	28	7
PRODUCER 3	86	129	61	208
PRODUCER 4	46	24	38	189
PERIOD 2				
PRODUCER 1	86	115	70	199
PRODUCER 2	40	159	32	8
PRODUCER 3	84	126	59	202
PRODUCER 4	46	24	38	189
PERIOD 3				
PRODUCER 1	84	113	69	196
PRODUCER 2	40	159	32	8
PRODUCER 3	84	126	60	203
PRODUCER 4	46	24	38	189
PERIOD 4				
PRODUCER 1	83	111	68	193
PRODUCER 2	41	163	33	8
PRODUCER 3	84	126	60	203
PRODUCER 4	46	24	38	189
PERIOD 5				
PRODUCER 1	82	110	67	190
PRODUCER 2	41	163	33	8
PRODUCER 3	85	127	60	205
PRODUCER 4	46	24	38	189

CHAPTER V
CONCLUSIONS AND FUTURE RESEARCH

The research described in is report contributes to the state-of-the-art of developing large-scale management information systems. An in-depth study has been reported describing the implementation of multi-level models for manpower planning. The report began with a review of the current status of such models in the Federal Government. This was followed by a detailed explanation of the strategic and tactical design considerations for the supporting applications software. Then, several numerical examples were given to show the comprehensiveness and management usefulness of the models.

The management outputs set the requirements for the supporting applications software designs of the multi-level model. Management outputs needed from the resource allocation submodel include reports to show: (a) differences between final user requirements and actual fulfillments and (b) producer resource availabilities in relation to the amount of resources which would be consumed. Reports needed from the manpower submodel include: (a) recruiting requirements data and (b) analysis of differences between the manpower requirements and the projected on-board population. In addition to the above, there is a need for reports to show the various input data which were used. Technical reports are also required for use by those who design and maintain the applications software.

The designs for the multi-level model applications software must take into account the variety of outputs which may be requested. There is a need for a considerable amount of generalized software such as was described in Chapter III. The generality of the FYNCIMP system has been carefully preserved by the use of tables and dictionaries. This allows the system to be used to process sequential sets of numbers rather than variable schemes which have been embedded in the computer programs themselves.

This research documents the way changes in the multi-level model applications software can be made in an on-going operational environment. It shows that the software designs must be modular to ensure that possible changes in the model mathematics can be incorporated without disrupting the operation of the system. This modularity is one of the features of the FYNCIMP system software design.

The applications software design which was presented in Chapter III includes a generalized model matrix generator and management reports writer. The generation of models is governed by a set of control cards and the data tapes containing the manpower transition rates and the manpower requirements data. The report writer uses a set of control cards to provide the English titles used on the management outputs. Some of the model types which can be generated include: (a) career management models with manpower and/or budgetary constraints, (b) average grade policy testing models, (c) multi-level models without manpower goals, and (d) multi-level models with manpower goals.

One of the underlying design provisions for the FYNCIMP system was to allow as much flexibility as possible until overcome by computer or administrative processing constraints. An example of this is the Transition Rate subsystem where data studies can be tailored by: (a) control procedures within the computer programs themselves, (b) extract programs to delimit the population to be processed, and (c) the possible use of several different formats of the input files. Flexibility is also obtained by the possible use of the occupational-grade level state dictionaries. These allow any combination of occupation and grades that fit into 100 occupation groups and five grade/level groupings.

Both the Transition Rate and Expected Retirement subsystem can be used to provide a set of historical data for management evaluation purposes. Data files are being maintained on a quarterly basis beginning with 30 June 1967 data. Several of the operational data studies which have been completed were for the specific purpose of making comparisons of changes in transition and retirement rates. These historical rates have also been used in various combinations to reflect better a future period which was being projected. This use of the transition rates was included as part of all three of the examples of the career management model which have been cited th's report. A new feature of the Transition Rate subsystem will include the ability to obtain personnel movements data by minority group. Studies comparing the promotion rates of women with the general population cou' . also be made.

Research studies have been made to examine the stability of the transition rates and of the retirement rates. A study of this type was the basis for the design change providing for the use of the retirement rate of a major occupation group for all members of that group. Similar studies have yielded the fact that there is considerable stability in the transition rates when categories with fifty or more people are used. This stability increases as the sample size increases.

Variety of usage has also been provided in the FYNCIMP system through the use of data change programs in appropriate points in the system. For example, the gross requirements data can be changed to include judgmental as well as algorithmic inputs. A change program is provided so that any coefficient in the transition matrix may be changed to reflect the underlying personnel movements of a particular policy alternative under study. There are also two points at which the linear programming matrix can be changed. This can be done either by the linear programming matrix generator change program or by use of the revision procedures in the linear programming software itself. Thus, all of the coefficients of the multi-level models can be changed at the point most efficient for computer processing. This is also done in such a manner as to ensure reasonable computer turnaround times.

The FYNCIMP system was designed with the user in mind. This includes an extensive management report generation capability. All of the data which have been encoded into alphanumeric code combinations for mathematical processing are decoded where possible back into English. The report formats are designed to pack most of the relevant information on as few pages as possible. This was done while at the same time trying to make the reports pleasing to the eye.

Many extensions are possible to the multi-level models. These include changes in the mathematical structures, applications software designs, data analysis techniques, and management practices suggested by these resource-manpower planning models. Many of the mathematical possibilities have already been conceptually enumerated. In addition, explicit model structural changes have been posited in the form of the chance-constrained extensions.

Another fundamental research area is the combination of these models with behavioral science techniques. The transition rate matrix elements have important management consequences when included in a model structure. The understanding of such consequences would best be done by behavioral scientists using the multi-level model as a research tool. Experiments could be established to obtain the insight that is needed to guide the organizational alternatives that are possible in changing the relative attractiveness of the different occupations. The model could be used to cost and evaluate these possible changes for potential managerial use. At the same time these behavioral science experiments could include the evaluation of career prospects for both potential and incumbent employees. Little research has been done with these and other possible behavioral science extensions to models of this kind.

Many experiments could be accomplished using the multi-level model applications software. One of these which is already underway is the conversational use of the models by means of remote computer terminals. These studies are being conducted by [illegible] son of Carnegie-Mellon University. The purpose of these studies is not to demonstrate the feasibility [illegible] versational computing. The important methodological question is the monitoring of user reactions to [illegible] e what procedures to use to overcome the natural use resistance to new managerial techniques such as the multi-level models. This means more than fitting the models into already existing perceptions of management. It also means altering in an experimental environment the management practices themselves.

Additional research possibilities include the development of more efficient model algorithms. For example, examination could be made of the possibilities of using smaller "driver" models to map out the basic management policies involved. This would then be followed up by the solution of a full-scale model which takes advantage of the advance start provided by the solution of the smaller driver problem. Examination could also be made of the use the multi-level model in a computer which has parallel central processors. Here, each of the sub-models could be solved in a separate parallel processor and then the results put together to obtain the overall system solution. Computer science research of this kind could be initiated in parallel with the continuing checkout of the models with operational data.

Implementation research such as described in this report should be continued and extended. It has to be remembered that the payoff of the research is the implementation and use of the multi-level models for management decisions by resource planners and manpower managers. This is best done through use of the applications software to support operational studies. In this way the peculiarities of on-going problems can be used to evaluate, shape, and extend the underlying multi-level models.

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PART II

AGGREGATIVE PLANNING MODELS

A GOAL PROGRAMMING MODEL FOR MANPOWER PLANNING

by

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Abstract

A goal programming model is formulated for guiding and controlling manpower planning at the level of the Office of Civilian Manpower Management of the U. S. Navy. Markov elements are used to trace through the effects of initial and subsequent personal commitments and budgeting constraints, personnel ceilings, etc., form parts of the total (Multi-dimensional) goals considered. Further extensions will include training, environmental factors, etc., after clarification is secured concerning the pertinence of such a line of development.

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1. Introduction

This report should be regarded as the first in a series of technical reports directed toward the erection of manpower planning and control models for the Office of Civilian Manpower Management (OCMM) in the U. S. Navy. It should also be borne in mind that this is only one part of a three-pronged program that involves: (i) Preparation of a conceptual paper which will provide (a) a statement of the manpower-management problem at OCMM and (b) a statement of objectives, as well as (ii) Development of a systems blueprint which will tie the elements of the problem together with the information requirements and decision possibilities and (iii) Provision for a continuing research effort pointed toward a synthesis of suitable models for implementing the latter (i.e., system) results in the light of the stated problems and objectives.[1]

It was recognized that there need not be any delay in carrying out any part of this program pending completion of the other portions. Provided contact and coordination could be maintained between the persons who were involved, something might be gained, in fact, by utilizing developments in each phase so that they could interact with and clarify issues in the other two phases.

This, then, is an objective of the present report. More precisely, we propose to introduce a series of technical (mathematical) developments which will help to clarify some of the possibilities that are now present. This should be regarded, however, as only a beginning for such possibilities. It does not represent a final commitment.

In addition to the opportunity that this presents for exploring the possibilities for modeling, we have tried to present this report so that it will help to clarify issues for the proposed systems blueprint as well. For instance, we have provided a numerical illustration in order to help focus on issues like data availability, forecasting and estimation requirements, etc. We have not undertaken the further mathematical research that would be needed for the development of solution procedures, or even more compact (and elegant) representations.[2] As of the moment, it seems better to focus on other issues which are best served, we believe, by restricting attention to modelling possibilities and systems synthesis at OCMM.

Although the numerical illustration is grossly oversimplified, we should probably also make clear that the proposed model already differs in some respects from others which are now available. To be sure, it incorporates Markovian elements, as do many other models, but these are only part of a total model of goal-programming variety[3] which lends itself to multiple-objective and multiple-criteria possibilities such as need to be considered when dealing with the multi-faceted complexities that are likely to be involved in personnel planning. As a planning model, this one is designed to provide a choice among all possible alternatives in filling vacancies from within, from training, and from outside sources in accordance with staged goals.

These and other features of the model can best be exhibited, however, by proceeding toward a mathematical development such as will now be undertaken in the immediately following sections.

2. Development of the Constraints

We shall normally be concerned with a sequence of periods, which can be indexed by $t = 0, 1, 2, \ldots,$ so that we can represent, say,

$$x_{ij}(t) = \text{number of personnel assigned to "job type i" from "}j^{th}\text{ source" for period t.} \qquad (1)$$

[1] Summarized from [34.1 a].

[2] Possible directions for such further research are presented in the body of the report and the model, and related developments, are carried to a point where the reasons for the suggested directions should be sufficiently clear.

[3] See [3] Appendix B and Chapter X for discussions of goal programming and related developments.

The terms in quotation marks may be singled out for clarification by observing that different "job types" can refer to the same job classification when the latter requires further distinction by "claimant" or "activity." Similarly, "sources" can also be subdivided and identified with different indexes arranged according to geographic or other characteristics when desired.[1] Thus, to accommodate such distinctions we introduce the set of indices defined by

$$J_{o\alpha} \equiv \{j: \text{ from "outside source" } \alpha\}$$

and (2.1)

$$J_o \equiv \sum_{\alpha} J_{o\alpha}$$

For instance, we might associate blue collar sources with $\alpha = 1, \ldots, \alpha_1$ and white collar sources with $\alpha = \alpha_1 + 1, \ldots, \alpha_2$. Similarly we might define more generally

$$J_{s\alpha} \equiv \{j: \text{ from } s^{th} \text{ category of source } \alpha\}. \tag{2.2}$$

For instance $s = o$, as in (2.1) denotes "outside sources" while other values assigned to s might be associated with retraining or other such source possibilities.

Now we introduce the following additional symbols and definitions,

$$\begin{aligned} c_i(t) &= \text{salary for } i^{th} \text{ job type in period t} \\ f_k(t) &= k^{th} \text{ manpower ceiling in period t} \\ I_k &= \{\text{jobs i under the } k^{th} \text{ manpower ceiling}\}, \end{aligned} \tag{3}$$

where we observe that I_k refers to the *set* of jobs, as indicated. Note that the $c_i(t)$ and $f_k(t)$ are prescribed values, as distinguished from the $x_{ij}(t)$ which are to be chosen in order to optimize the planning objectives. Initially at least, these $c_i(t)$ and $f_k(t)$ values may be obtained by means of estimates or forecasts as well as from stipulated policies or regulations. However, the model to be erected will allow for evaluating the consequences of varying these $f_k(t)$ and $c_i(t)$ so that guidance for manpower planning may be secured from this quarter as well.

We can explicitly exhibit the relation between the $x_{ij}(t)$ and $f_k(t)$ by means of the following expression for the discrepancy between scheduled manpower attainments and the manpower ceilings,

$$\sum_j \sum_{i \epsilon I_k} x_{ij}(t) - f_k(t) \equiv E_k(t). \tag{4}$$

Clearly, $E_k(t)$ may be positive, negative or zero. That is, we permit some violation of this $f_k(t)$ ceiling, which we can control further, if desired, by prescribing further constraints on the permitted limits for these discrepancies.[2]

In the future we may also need some refinement of the variables $x_{ij}(t)$ in order to designate their applicability to particular programs. Suppose, for instance, that program r in period t needs at least $p_{ir}(t)$ men in job i. Then we shall need to write

$$x_{ij}(t) = \sum_r x_{ij}^r(t) \tag{5}$$

[1]Cf., e.g., the distinction between "East Coast Welders" and "West Coast Welders" in the numerical illustration (below, section 5).

[2]See, e.g., Appendix B and Chapter X in [3].

where $x_{ij}^r(t)$ designates that part of $x_{ij}(t)$ allocated to program r. This manpower requirement for program r could then be written

$$\sum_j x_{ij}^r(t) \geq p_{ir}(t). \tag{6}$$

Similarly, we may require a $b_r(t)$ to designate the dollar budget which is applicable to program r in period t. And we may need variables $w_{ir}(t)$ to designate the chosen manpower reduction in job i of program r.

In this initial model we shall restrict ourselves to consideration of manpower planning in which our manpower sources are twofold: (1) from within the organization and (2) from outside sources.[1] Let us suppose that the changes and attrition from one job to another within the organization are given by the Markoff matrix M with element $M_{i\ell}$ where $M_{i\ell}$ represents the proportion of those in job ℓ in the previous period who will go to job i in the current period. In this Markoff representation, attrition or loss of manpower is represented by

$$\sum_i M_{i\ell} < 1, \tag{7}$$

i.e., some of the persons in job ℓ are not retained in any job i.

If

$$a_i \equiv \text{number of persons in job i initially (i.e., at } t = 0) \tag{8}$$

then

$$\sum_\ell M_{i\ell} a_\ell = \text{number of persons in job i in period 1 from within.} \tag{9}$$

The total in job i in period 1 is then given by

$$\sum_\ell M_{i\ell} a_\ell + \sum_{j \epsilon J_o} x_{ij}(1) \tag{10}$$

where, as before, $J_o = \{j\text{: from outside sources}\}$. The number in job i in period 2 from within is given by

$$\sum_p M_{ip} \left(\sum_\ell M_{i\ell} a_\ell + \sum_{j \epsilon J_o} x_{pj}(1)\right). \tag{11}$$

At this point it is convenient to convert to matrix notation and so we introduce the following definitions:

$$x^j(t) = \begin{pmatrix} x_{1j}(t) \\ \vdots \\ x_{ij}(t) \\ \vdots \\ x_{nj}(t) \end{pmatrix} \qquad a = \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix} \tag{12}$$

$$(M)_i = (M_{i1}, \ldots, M_{i\ell}, \ldots, M_{in})$$

[1] i.e., we do not treat retraining, etc., explicitly in this first model.

Hence, in matrix notation, the number in job i in period 2 from within is

$$(M^2)_i\, a + \sum_{j\epsilon J_o} (M)_i\, x^j(1) \tag{13}$$

and the *total* in period 2 is

$$(M^2)_i\, a + \sum_{j\epsilon J_o} (M)_i\, x^j(1) + \sum_{j\epsilon J_o} x_{ij}(2). \tag{14}$$

Thus, in general, the total in job i in period t is

$$(M^t)_i\, a + \sum_{\tau=1}^{t} \sum_{j\epsilon J_o} (M^{t-\tau})_i\, x^j(\tau). \tag{15}$$

For this initial model we elect to stay within a total dollar budget that is stipulated for each period. Thus, if

$$B(t) \equiv \text{total \$ budget for personnel in period t}, \tag{16}$$

then we shall require the $x_{ij}(t)$ values to satisfy

$$\sum_i \sum_j c_i(t)\, x_{ij}(t) \leqslant B(t). \tag{17}$$

Note that j here runs over J_o and also another index which corresponds to the source "within the organization."

3. **Representation of Model and Objectives**

We shall here formulate an objective by supposing that we wish to minimize the discrepancies, as given in (4). I.e., we propose to minimize

$$\sum_k \sum_t \mu_{kt}\, |E_k(t)|, \tag{18}$$

where the μ_{kt} are weights associated with the k^{th} manpower ceiling and the vertical strokes denote absolute values.

This nonlinear objective function may be reduced to more tractable form by utilizing the theory and procedures that we have developed in connection with other personnel planning models.[1] Thus we introduce the new variables $E_k^+(t), E_k^-(t) \geqslant 0$ and then represent

$$E_k(t) = E_k^+(t) - E_k^-(t). \tag{19}$$

The objective in (18) can then be represented by

$$\min \sum_k \sum_t \mu_{kt} (E_k^+(t) + E_k^-(t))$$

with

$$\sum_{i\epsilon I_k} \sum_{\mu=1}^{t} \sum_{j\epsilon J_o} (M^{t-\tau})_i\, x^j(\tau) - E_k^+(t) + E_k^-(t) = g_k(t) \tag{20}$$

[1] Cf. the discussion in Chapter X of [3].

where

$$g_k(t) = f_k(t) - \sum_{i \in I_k} (M^t)_i\, a.$$

In other words, $g_k(t)$ is the net k^{th} manpower requirement which must be met, if at all, by outside recruitment.

In a similar manner, substituting for the $x_{ij}(t)$, the budget constraints become

$$\sum_i \sum_{\tau=1}^{t} \sum_{j \in J_o} c_i(t)\,(M^{t-\tau})_i\, x^j(\tau) \leqslant B(t) - \sum_i c_i(t)\,(M^t)_i\, a \tag{21}$$

with

$$x^j(\tau) \geqslant 0.$$

Thus the model may be represented

$$\sum_k \sum_t \mu_{kt}\, [E_k^+(t) + E_k^-(t)]$$

subject to

$$\sum_{i \in I_k} \sum_{\tau=1}^{t} \sum_{j \in J_o} (M^{t-\tau})_i\, x^j(\tau) - E_k^+(t) + E_k^-(t) = g_k(t)$$

$$\sum_i \sum_{\tau=1}^{t} \sum_{j \in J_o} c_i(t)\,(M^{t-\tau})_i\, x^j(\tau) \leqslant B(t) - c^T(t)\, M^t a \tag{22}$$

$$x^j(\tau) \geqslant 0$$

$$E_k^+(t),\ E_k^-(t) \geqslant 0$$

where

$$c^T(t) \equiv (c_1(t), \ldots, c_n(t)).$$

4. Transformation and Reduction of the Model

The model could be calculated as represented above. We note, however, that it is possible to simplify it by making certain transformation of the variables since the basic variables $x^j(t)$ only enter in certain combinations. Thus, let

$$\xi(\tau) = \sum_{j \in J_o} x^j(\tau)$$

$$\eta(t) = \sum_{\tau=1}^{t} M^{t-\tau}\, \xi(\tau). \tag{23}$$

Then

$$\eta(t+1) = \sum_{\tau=1}^{t+1} M^{t+1-\tau}\,\xi(\tau)$$

$$= M \sum_{\tau=1}^{t} M^{t-\tau}\,\xi(\tau) + \xi(t+1) \tag{24}$$

$$= M\,\eta(t) + \xi(t+1).$$

Note that since $\xi(t+1)$ are vectors of decision variables and are non-negative, we can replace them by the choice vectors $\eta(t)$ with the requirement

$$\eta(t+1) - M\,\eta(t) \geqslant 0$$

and

$$\eta(t) \geqslant 0 \tag{25}$$

The model can now be represented

$$\min \sum_k \sum_t \mu_{kt}\,[E_k^+(t) + E_k^-(t)]$$

subject to:

$$\sum_{i \in I_k} \eta_i(t) - E_k^+(t) + E_k^-(t) = g_k(t)$$

$$\sum_i c_i(t)\,\eta_i(t) \leqslant B(t) - c^T(t)\,M^t a$$

$$-(M)_i\,\eta(t) + \eta_i(t+1) \geqslant 0$$

$$\eta_i(t),\ E_k^+(t),\ E_k^-(t) \geqslant 0$$

where $\eta_i(t)$ is the i^{th} component of $\eta(t)$.

Schematically, the non-zero coefficients may be arrayed as in Figure 1, below.

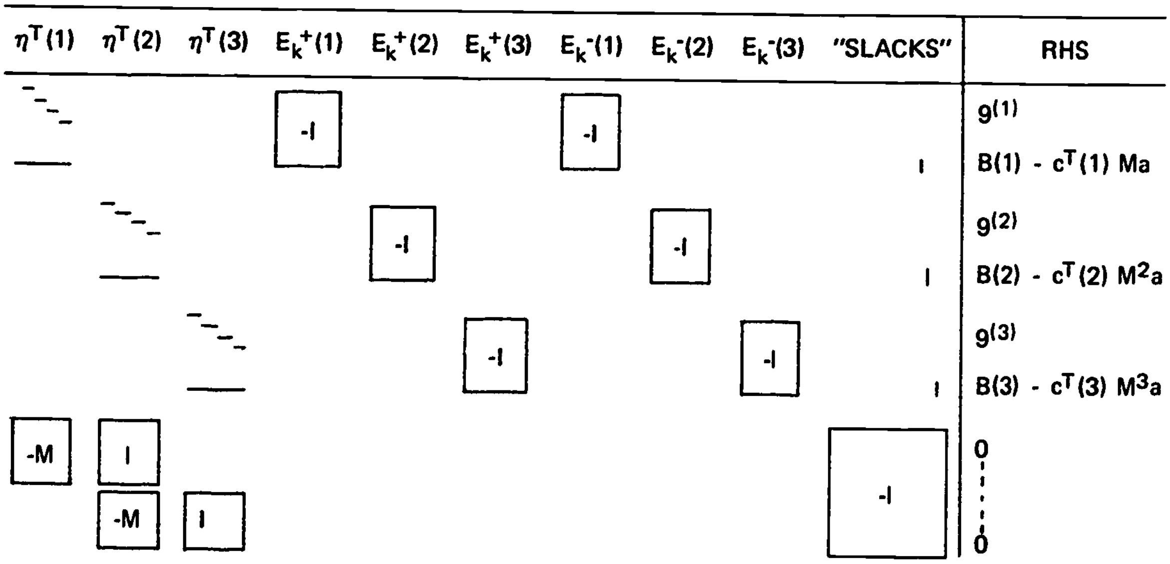

Figure 1. ARRAY OF NON-ZERO COEFFICIENTS

5. Numerical Illustration

In order to make the preceding developments somewhat more concrete, we now proceed to a numerical illustration. This is the only purpose of the illustration, however, since the data are all purely hypothetical and contrived for a highly simplified two-period categorization. To emphasize this even further, we refrain from pushing on to a solution of the resulting model since this would be of no interest per se. Instead, we shall discuss some of the computation possibilities as well as the evaluation and other possibilities for an integrated personnel planning system—as these might be attended to by further research.

For illustrative job types we shall utilize the following:

i	Description	Abbreviation
1	Personnel Analyst	PA
2	Mechanical Engineer	ME
3	Welder—West Coast	WC
4	Welder—East Coast	EG

The transition probabilities and the related Markoff matrix, M, which we shall use in this illustration is arranged so that the transitions[1] are from ℓ to i. Using blank cells to represent $M_{i\ell} = 0$, we suppose M to be

ℓ / i		PA 1	ME 2	WC 3	EC 4
PA	1	.8	.1		
ME	2	.1	.7		
WC	3			.6	
EC	4			.1	.9

Restricting our illustration to only two periods, we represent the $g_k(t)$ values by[2]

k / t	1	2	3	4
1	30	200	600	500
2	70	300	450	500

Next, we hypothesize some representative salary values in which we suppose that $c_i(1) = c_i(2)$ for all i. Stating these hypothesized values in units of $1,000 we have

c_1	c_2	c_3	c_4
15	13	8	7

[1] I.e., the $M_{i\ell}$ represent the transition rates from ℓ to i in each cell M.

[2] If wanted the $f_k(t)$ values may be obtained from

$$f_k(1) = g_k(1) + (M)_k\, a$$

$$f_k(2) = g_k(2) + (M^2)_k\, a$$

Stipulated budget ceilings are also stated in units of $1,000 for each period as

B (1)	B (2)
17,800	16,900

Initial a_i values are components for the vector a of personnel already in position at the start are

PA	ME	WC	WE
25	220	550	450

Thus to obtain the value for $c^T(1)$ M a and $c^T(2)$ M^2 a we obtain

$$c^T M = (13.3, 10.6, 5.5, 6.3)$$

which can be multiplied by the corresponding components of a, and summed, to yield

$$c^T(1) M a = c^T M A = 8540$$

and by a similar route we obtain

$$c^T(2) M^2 a = c^T M^2 a = 6930$$

Referring back to the data for dollar budget ceilings we thus obtain

$$B(1) - c^T M a = 17{,}800 - 8{,}540 = \$9{,}260$$

$$B(2) - c^T M^2 a = 16{,}900 - 6{,}930 = \$9{,}970.$$

Positioning these data in the form suggested by Figure 1, we obtain the concrete representation that is depicted in Figure 2.

It may be noted from the above figure that a method of "model approximation" may be used which is analogous to the one developed for the pipeliner example in association with the oil field development programming undertaken by ARAMCO.[1] Thus, in Figure 1, if the first sets of unknowns ($\eta^T(1)$ then $\eta^T(2)$) had values substituted for them, then these sets of equations would reduce to individual lower-bound inequalities on $\eta^T(2)$ and $\eta^T(3)$. The resulting structure by changes of scale and variables and multiplication of equations by suitable constants would be reduced to a very special form of the distribution (or transportation) model which could be immediately brought into contact with the highly efficient algorithms that are available for these classes of models.[2] The same type of parametric procedure as in the pipeliner model could then be used to obtain an optimal exact solution.

Before work of the above type is undertaken, however, it is prudent to consider ways in which the model might itself be altered or extended. Training facilities and environmental factors have already been noted as candidates for explicit treatment and this does not exhaust the list. A strategy for staging such further developments might thus also form a part of the topics to be considered after the model presented above is reviewed in the context of the 3-pronged OCMM program which was discussed at the outset of this report.

[1] See e.g., Chapter XVI in [3].
[2] Cf., e.g., Chapter II and XIV in [3].

η_1 (1)	η_2 (1)	η_3 (1)	η_4 (1)	η_1 (2)	η_2 (2)	η_3 (2)	η_4 (2)	$E_1^+(1)$	$E_2^+(1)$	$E_3^+(1)$	$E_4^+(1)$	$E_1^+(2)$	$E_2^+(2)$	$E_3^+(2)$	$E_4^+(2)$	E_1 (1)	E_2^- (1)	E_3^- (1)	E_4^- (1)	E_1^- (2)	E_2^- (2)	E_3^- (2)	E_4^- (2)	s_1^-	s_2^-	s_3^-	s_4^-	s_1^+	s_2^+	RHS
1								-1								1														30
	1								-1								1													200
		1								-1								1												600
			1								-1								1											500
15	13	8	7																									1		9260
				1								-1								1										70
					1								-1								1									300
						1								-1								1								450
							1								-1								1							500
				15	13	8	7																						1	9970
-.8	-.1			1																				-1						0
-.1	-.7				1																				-1					0
		-.6				1																				-1				0
		-.1	-.9				1																				-1			0

Figure 2

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34.1 Memo to Files from A. Charnes and W. W. Cooper

a. Memo of June 15, 1967 covering minutes of a conference with Mr. Robert Willey, Captain P. A. Gisvold, Mr. R. H. Rawdon, Mr. W. N. Price and Lt. R. J. Niehaus.

b. Memo of October 13, 1967, covering review of proposed model with Mr. Robert Willey, Captain P. A. Gisvold, Mr. P. Meyerson, Mr. R. H. Rawdon, Lt. R. J. Niehaus, Mr. W. Price and Mr. J. Treires. Note, this includes Memo of 10 October 1967 prepared by Lt. R. J. Niehaus and revised in association with A. Charnes and W. W. Cooper covering (i) June 15, 1967 meeting with Mr. Willey, (ii) development of a Markov model program and (iii) description of this plus the model developed in the meeting of Niehaus, Charnes and Cooper in Pittsburgh on 2 August 1967 as well as (iv) description of this and other research under way at that time.

c. Memo of December 4, 1967 covering meeting of Lts. R. J. Niehaus and David Sholtz with Professors A. Charnes and W. W. Cooper in Pittsburgh on 12/4–12/5/67. Rough notes only, to cover redo of model in notes of 8/2/67.

d. Memo of August 2, 1967 covering meeting of Lt. R. J. Niehaus and Professors A. Charnes and W. W. Cooper. Rough notes only, to record model details during meetings of 8/1–8/2/67.

e. Memo of 29 June 1967 covering meeting with W. N. Price, R. H. Rawdon, R. J. Niehaus and P. Weinstein. Review of relation between concept and systems work and modelling research. See also memo of 7 July 1967 prepared by R. J. Niehaus.

f. Memo of 16 May 1967 covering results of field trip to Portsmouth Shipyard on May 7, 8, 9, 1967 and discussions with Admiral Hushing and Donald Holster (and others) on WOWMANS scheduling by Lt. Niehaus in association with Professors A. Charnes and W. W. Cooper.

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A MODEL FOR CIVILIAN MANPOWER MANAGEMENT AND PLANNING IN THE U. S. NAVY

by

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D. Sholtz

Abstract

Previous developments of manpower planning models involving uses of goal programming with embedded Markoff processes are here extended in order to (a) explicitly comprehend truncational effects—e.g., those due to retirement—and (b) allow for interperiod Markoff transition matrices which change over time. A prototype example is included to show a solution of the original goal programming model using "live" data.

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I. Introduction

This paper reports on some further developments in a research program being undertaken in cooperation with OCMM (Office of Civilian Manpower Management). This part of the OCMM effort looks toward improving the processes of manpower planning for the U. S. Navy by means of computer assisted mathematical models. See, e.g., [4]-[8] and [11] in the bibliography, which reference earlier papers that extended the "state of the art" for modelling manpower planning by (a) combining the ideas of "goal programming" and Markoff transition processes and (b) utilizing multiple objectives along with (c) other constraints—such as budgetary limitations—so that these related considerations could also be simultaneously treated along with the personnel programs (and proposals) to which they might be related.[1]

In this report, these previously developed models are further extended and refined to explicitly comprehend truncational effects (e.g., those associated with retirement) as well as the possibility of interperiod Markoff transition matrices which change over time. It is then also possible, as we shall soon see, to extend the transformational formulas that were found advantageous for simplifying and improving computations for the original model. Furthermore, although only one particular device for handling truncations is presented here, it will be seen that others can also be used in addition to the "locked-in" truncations that we shall depict in this report. (It is important to observe this kind of possibility especially in connection with the many byproduct uses that the extended model is designed to yield.)

All of the above research is specifically designed to tie in with the automated civilian manpower information systems that the U. S. Navy is developing. These models and their related developments are, as will also be seen, applicable in other contexts as well.

II. Recapitulation of OCMM's Goal Programming Model for Manpower Planning

For ease of reference we recapitulate and summarize relevent details of from the already developed model which we have previously reported. See, e.g., [4] through [7]. Thus we write

$$\min \sum_{k} \sum_{t} \mu_{kt} \left(E_k^+(t) + E_k^-(t)\right)$$

subject to

$$\sum_{i \epsilon I_k} \sum_{\tau=1}^{t} \sum_{j \epsilon J_o} (M^{t-\tau})_i x^j(\tau) - E_k^+(t) + E_k^-(t) = g_k(t) \tag{1}$$

where

$$g_k(t) = f_k(t) - \sum_{i \epsilon I_k} (M^t)_i\, a. \tag{1.1}$$

and $f_k(t)$ is the ceiling prescribed for the k^{th} manpower category. The subtraction from $f_k(t)$ in (1.1) yields a $g_k(t)$ value that represents the net requirement for the k^{th} manpower category after allowance for the

[1]See [3] for further detailed development on the ideas of goal programming, multiple objectives, etc. Possibilities for policy evaluations and sensitivity analysis, as well as various byproducts of value for personnel planning are also examined in [4], [5], [6], and [7].

available persons remaining in this category through the growth and attrition which occurred during the t previous periods, where

$$a = \text{vector of initial inventory of personnel in all job types}$$
$$M = \text{Markoff transition matrix depicting the transfer probabilities between job types} \tag{2}$$

while $(M^t)_i$ = the i^{th} row of M^t and $M^t = MM \ldots M$ (t times). Thus

$$\left[\sum_{i \epsilon I_k} (M^t)_i \, a\right]$$

represents the carry-forward of the initial states as they pertain to k^{th} manpower category comprised of the collection I_k of job types i.

The above model does not contain budgetary or other constraints and hence we can look at it for purely manpower planning aspects in a model where the objective is to minimize a weighted sum of the deviations for each manpower category via

$$E_k^+(t),\ E_k^-(t) = \text{positive or negative deviation, respectively, for } k^{th} \text{ manpower category.}$$
$$\mu_{kt} = \text{the weight which is applicable to } k^{th} \text{ manpower category in period t.} \tag{3}$$

Using B(t) to represent an applicable budgetary ceiling for expenditures in period t and

$$c^T(t) = (c_1(t), \ldots, c_i(t), \ldots, c_m(t))$$

as a vector of salary rates, we can extend the model in (1) by adjoining constraining budgetary relations as follows:

$$\min \sum_k \sum_t \mu_{kt} \left[E_k^+(t) + E_k^-(t)\right]$$

subject to

$$\sum_{i \epsilon I_k} \sum_{\tau=1}^{t} \sum_{j \epsilon J_o} (M^{t-\tau})_i \, x^j(\tau) - E_k^+(t) + E_k^-(t) = g_k(t)$$
$$\sum_i \sum_{\tau=1}^{t} \sum_{j \epsilon J_o} c_i(\tau) \, (M^{t-\tau})_i \, x^j(t) \leq B(t) - c^T(t) \, M^t a \tag{4}$$

where J_o refers to the collection of external manpower sources and $x^j(t) \geq 0$ refers to the vector of amounts secured from source $j \epsilon J_o$ in period t.

As was noted in [4], the basic variables $x^j(t)$ only enter in certain combinations. Thus it is possible to simplify the model in (4) by making certain transformations for which the following symbols (and related definitions) may be utilized:

$$\xi(\tau) = \sum_{j \epsilon J_o} x^j(\tau)$$
$$\eta(t) = \sum_{\tau=1}^{t} M^{t-\tau} \, \xi(\tau) \tag{5}$$

The pertinent parts of (4) can then become simplified via the expressions in (5)—viz.,

$$\sum_{i\epsilon I_k} \sum_{\tau=1}^{t} \sum_{j\epsilon J_o} (M^{t-\tau})_i \, x^j(\tau) = \sum_{i\epsilon I_k} \sum_{\tau=1}^{t} (M^{t-\tau})_i \, \xi(\tau) = \sum_{i\epsilon I_k} (\eta(t))_i. \tag{6}$$

Furthermore, we can utilize the definition $M^o \equiv I$ (the identity matrix) to obtain

$$\sum_{i\epsilon I_k} \sum_{\tau=1}^{t+1} (M^{t+1-\tau})_i \, \xi(\tau) = \sum_{i\epsilon} \sum_{\tau=1}^{t} M^{t+1-\tau} \, \xi(\tau) + M^o \, \xi(t+1) = \sum_{i\epsilon I_k} M \sum_{\tau=1}^{t} M^{t-\tau} \, \xi(\tau) + \xi(t+1). \tag{7}$$

Hence, we also have

$$\eta(t+1) = M\eta(t) + \xi(t+1). \tag{8}$$

Since the $\xi(t+1)$ are vectors of decision variables and are non-negative, we can proceed as in [4] to replace them by the choice vectors $\eta(t)$ with the requirement

$$\eta(t+1) - M\eta(t) \geqslant 0$$

and (9)

$$\eta(t) \geqslant 0.$$

Preparatory to the interpretations and elaborations undertaken in the next section, we can utilize these developments to replace the original model in (4) by the following transformed and reduced version—*viz*..

$$\min \sum_k \sum_t \mu_{kt} \left[E_k^+(t) + E_k^-(t)\right]$$

subject to:

$$\begin{aligned} &\sum_{i\epsilon I_k} \eta_i(t) - E_k^+(t) + E_k^-(t) = g_k(t) \\ &\sum_i c_i(t)\, \eta_i(t) \leqslant B(t) - c^T(t)\, M^t a \\ &- (M)_i \, \eta(t) + \eta_i(t+1) \geqslant 0 \\ &\eta_i(t),\ E_k^+(t),\ E_k^-(t) \geqslant 0. \end{aligned} \tag{10}$$

III. Interpretations and Further Development

To interpret what has been achieved above, note first that $\eta_i(t)$ is the i^{th} component of $\eta(t)$ where

$$\eta(t) \equiv \text{Accumulated inventory from hiring to period t.} \tag{11}$$

The vector $\eta(t)$ can be related to the recruitment that will be programmed by means of the vector $\xi(t)$ and other relations as follows. First, the amount to be recruited in period 1 is given by

$$\eta(1) = \xi(1)$$

where $\xi(1)$ is the amount to be recruited for period 1. (See (5). For period 2, however,

$$\eta(2) = M\xi(1) + \xi(2)$$

where $\xi(2)$ is the amount to be recruited in period 2 and M is the Markoff matrix which has as its elements the probabilities of movement into and out of the various jobs. See (2). Of course, $(M)_i$ is the i^{th} row of M and

$$(M)_i = (M_{i1}, \ldots, M_{i\ell}, \ldots, M_{in})$$

relates this row to the elements $M_{i\ell}$ of M. The expression which relates the inventory of hirings in period 3 to the hiring activities up through this period is

$$\eta(3) = M^2\ \xi(1) + M\ \xi(2) + \xi(3),$$

and so on.

This leads to a "triangular system" which may be developed for convenient representation and solution by means of the following definitions

$$\begin{bmatrix} I & O \\ -M & I \end{bmatrix} \begin{bmatrix} I & O \\ M & I \end{bmatrix} = \begin{bmatrix} I & O \\ X+M & I \end{bmatrix}$$

where (12)

I = Identity matrix

X = -M.

Thus, for the 2 period case:

$$\begin{bmatrix} I & O \\ M & I \end{bmatrix} \begin{bmatrix} \xi(1) \\ \xi(2) \end{bmatrix} = \begin{bmatrix} \eta(1) \\ \eta(2) \end{bmatrix}$$

and

$$\begin{bmatrix} \xi(1) \\ \xi(2) \end{bmatrix} = \begin{bmatrix} I & O \\ M & I \end{bmatrix}^{-1} \begin{bmatrix} \eta(1) \\ \eta(2) \end{bmatrix}$$

$$\equiv \begin{bmatrix} I & O \\ M & I \end{bmatrix} \begin{bmatrix} \eta(1) \\ \eta(2) \end{bmatrix}$$

since—see (12) the indicated inverse exists.

Similarly for 3 periods

$$\begin{bmatrix} \xi(1) \\ \xi(2) \\ \xi(3) \end{bmatrix} = \begin{bmatrix} I & O & O \\ M & I & O \\ M^2 & M & I \end{bmatrix}^{-1} \begin{bmatrix} \eta(1) \\ \eta(2) \\ \eta(3) \end{bmatrix} \equiv \begin{bmatrix} I & O & O \\ -M & I & O \\ O & -M & I \end{bmatrix} \begin{bmatrix} \eta(1) \\ \eta(2) \\ \eta(3) \end{bmatrix}$$

or

$$\xi(1) = \eta(1)$$

$$\xi(2) = \eta(2) - M\eta(1)$$

$$\xi(3) = \eta(3) - M\eta(2),$$

and so on.

More generally, we can write

$$\xi(t) = \eta(t) - M\eta(t\text{-}1). \tag{13}$$

Pausing and reflecting on this structure we see that it can be significantly generalized to the situation where the Markoff matrices may differ from period to period. T s if $\eta(t)$ is accumulating through period t we can write

$$\xi(t) = \eta(t) - M\begin{pmatrix} t\text{-}1 \\ t \end{pmatrix} \eta(t\text{-}1) \tag{14}$$

where $M\begin{pmatrix} t\text{-}1 \\ t \end{pmatrix}$ is the probability matrix applying to the transitions from period t-1 to 1. (Note that the inversion is still straightforward in any case and requires no division operations, whether M is non-singular or not.)

IV. Adjustments for Variations in Proportions Eligible for Retirement

Part One: Estimation and Adjustment Formulas

We now develop a procedure for further extending the OCMM goal programming model—or any other such model—and its application by developing certain formulas which make better allowance for potential retirements. The nature of the approach to be suggested will be sufficiently clear, we think, if we proceed by reference to a five-period example by means of which we can provide more specific and concrete interpretations.

Those who are, or will be, eligible to retire in the next five years are represented by the vector b,

$$b = b^5 + b^4 + b^3 + b^2 + b^1 + b^0$$

where

b^5 = those *first* eligible to retire in period 5

b^4 = those *first* eligible to retire in period 4

b^3 = those *first* eligiole to retire in period 3

b^2 = those *first* eligible to retire in period 2 (15)

b^1 = those *first* eligible to retire in period 1

b^0 = those eligible to retire in period 0, but did not do so.

Next, let $a = \bar{a} + b$ where a = total "on board" at t = 0, $\bar{a}$ = those on board *not* eligible to retire in the next five years and b = those on board who are eligible to retire in the next five years.

Now we introduce the following expressions and definitions:

$$\alpha \equiv C_o/B_o, \quad \bar{\alpha} = (1 - C_o/B_o)$$

where

$$\begin{aligned} C_o &= \text{Personnel remaining from those eligible to retire in some base period} \\ B_o &= \text{Total eligible to retire in some base period.} \end{aligned} \tag{16}$$

For those who are eligible to retire in the next five periods, we shall assume—that either they will retire or that they will remain in the same job. Then we develop our formulas for what we shall call a five-period "stretch." This is done by period, as follows, where the formula for estimating those who will *not* retire is shown first in each of the five periods:

Period 1:

$$\bar{\alpha}\,(b^1 + b^0)$$

$$\alpha\,(b^1 + b^0)$$

Period 2:

$$\bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b)$$

$$\alpha b^2 + \alpha\bar{\alpha}(b^1 + b^0)$$

Period 3:

$$\bar{\alpha}b^3 + \bar{\alpha}\,[\bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b^0)] \tag{17}$$

$$\alpha b^3 + \alpha\,[\bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b^0)]$$

Period 4:

$$\bar{\alpha}b^4 + \bar{\alpha}\,[\bar{\alpha}b^3 + \bar{\alpha}^2b^2 + \bar{\alpha}^3(b^1 + b^0)]$$

$$\alpha b^4 + \alpha\,[\bar{\alpha}b^3 + \bar{\alpha}^2b^2 + \bar{\alpha}^3(b^1 + b^0)]$$

Period 5:

$$\bar{\alpha}b^5 + \bar{\alpha}\,[\bar{\alpha}b^4 + \bar{\alpha}^2b^3 + \bar{\alpha}^3b^2 + \bar{\alpha}^4(b^1 + b^0)]$$

$$\alpha b^5 + \alpha\,[\bar{\alpha}b^4 + \bar{\alpha}^2b^3 + \bar{\alpha}^3b^2 + \bar{\alpha}^4(b^1 + b^0)]\,.$$

Finally, it also seems to be fair to assume that those who are recruited during this five-period stretch will not be eligible to retire during this stretch.

Part Two: Goal Adjustments

An extended model may be secured from (10) by using the preceding developments and assumptions to adjust for the indicated retirements. These "retirement adjustment" corrections will be reflected, of course, in the budget constraints as well as in the goals and objectives.

As was observed in Section III, the original model utilized the matrix, M, to project all of the initial manpower plus manpower hired during the "stretch."[1] We now propose to utilize our preceding development, however, to encompass the situations involving retirement projections. Thus, if we use $\hat{\eta}(t)$ to represent the cumulative manpower vector through period t, then we can represent as a sum of two vectors—*viz.*,

$$\hat{\eta}(t) = \eta^1(t) + \eta^2(t) \tag{18}$$

where $\eta^1(t)$ represents the manpower cumulation *not* affected by retirement and $\eta^2(t)$ represents the cumulation affected by retirement. Furthermore, by an evident extension of (13) joined to the development in Part One of this section,

$$\begin{aligned} \eta^1(t) &= M\eta^1(t\text{-}1) + \xi(t) \\ \eta^2(t) &= \bar{\alpha}\eta^2(t\text{-}1) + \bar{\alpha}b^t. \end{aligned} \tag{19}$$

Specifically,

$$\begin{aligned} \eta^2(1) &= \bar{\alpha}(b^1 + b^0) \\ \eta^2(2) &= \bar{\alpha}b^2 + \bar{\alpha}\eta^2(1) \\ &= \bar{\alpha}b^2 + \bar{\alpha}^2(b^1 + b^0) \end{aligned} \tag{19.1}$$

as required for period 2—see (17)—and so on.

Now we let

$$a_i = \text{those in job i initially (at } t = 0) \tag{20}$$

and

$$a_i = \bar{a}_i + b_i, \tag{21}$$

so that, as in the development which immediately follows (15), we have $\bar{a}_i$ representing those *not* eligible for retirement in the five-period stretch covered by[2]

$$b = b^0 + b^1 + \ldots + b^5. \tag{22}$$

The vector of personnel in period 1 coming from within is

$$M\,\bar{a} + \bar{\alpha}\,(b^0 + b^1) = M\,\bar{a} + \eta^2(1), \tag{23}$$

via the first expression in (19.1). Similarly, the vector of personnel in period 2 coming from within is

$$M^2\,\bar{a} + \bar{\alpha}\,b^2 + \bar{\alpha}^2\,(b^0 + b^1) = M^2\,\bar{a} + \eta^2(2), \tag{24}$$

via the second expression in (19.1).

Now the total on board in period 1 equals

$$M\,\bar{a} + \eta^1(1) + \eta^2(1) + \xi(1) \tag{25}$$

[1] E.g., the five-period stretch which we are using.
[2] See (15).

where $\xi(1)$ represents hiring from outside while $M\,\bar{a}$ provides the transitions from the state given by $\bar{a}$, the remnant from $t = 0$. The vector of personnel from within at period 2 is

$$M^2\,\bar{a} + M\,\xi(1) + \eta^2(2) \tag{26.1}$$

which, in turn, forms a part of the total period 2 personnel

$$M^2\,\bar{a} + M\xi(1) + \eta^2(2) + \xi(2). \tag{26.2}$$

By an evident extension we can now write

$$M^t\,\bar{a} + \sum_{\tau=1}^{t} M^{t-\tau}\,\xi(\tau) + \eta^2(t) \tag{27}$$

as the expression for the vector of total personnel in period t. By virtue of the definition of $\eta^1(t)$ in (18),

$$\eta^1(t) = \sum_{\tau=1}^{t} M^{t-\tau}\,\xi(\tau) \equiv \xi(t) + \sum_{\tau=1}^{t-1} M^{t-\tau}\,\xi(\tau). \tag{28}$$

But,

$$\sum_{\tau=1}^{t-1} M^{t-\tau}\,\xi(\tau) = M \sum_{\tau=1}^{t-1} M^{(t-1)-\tau}\,\xi(\tau) \equiv \eta^1(t\text{-}1). \tag{29}$$

Thus, via (28) and (29),

$$\eta^1(1) = \xi(t) + \eta^1(t\text{-}1). \tag{30}$$

V. Adjustment and Extension of the Model

Now turning to the model (10), the objective function we wish to minimize is

$$\sum_k \sum_t \mu_{kt}\,(E_k^+(t) + E_k^-(t))$$

subject to goal (e.g., manpower ceilings) and budget constraints, where

μ_{kt} = weights associated with the k^{th} manpower ceiling in period t

$E_k^+(t)$ = excess above k^{th} goal in period t

$E_k^-(t)$ = deficiency below k^{th} goal in period t.

Both $E_k^+(t)$ and $E_k^-(t)$ will not appear in the solution at the same time since a given goal discrepancy can only be in one direction at any given time (e.g., if $E_k^+(t)$ is in the solution then $E_k^-(t)$ will equal zero).

Developing the goal constraints, in which there $E_k^+(t)$ and $E_k^-(t)$ variables appear, the k^{th} goal is defined as

$$\sum_{i \in I_k} \left[\text{Total personnel in period } t_i\right] - E_k^+(t) + E_k^-(t) = f_k(t)$$

where

$$f_k(t) = k^{th} \text{ manpower goal (or ceiling) in period } t.$$

Substituting the expression for total personnel in period t we have

$$\sum_{i \epsilon I_k} [(M^t\bar{a})_i + \eta_i^1(t) + \eta_i^2(t)] - E_k^+(t) + E_k^-(t) = f_k(t).$$

Rearranging, the goal constraint is

$$\sum_{i \epsilon I_k} \eta_i^1(t) - E_k^+(t) + E_k^-(t) = f_k(t) - \sum_{i \epsilon I_k} (M^t\bar{a})_i - \sum_{i \epsilon I_k} \eta_i^2(t) = \hat{g}_k(t)$$

where $\hat{g}_k(t)$ is the k^{th} manpower requirement obtained from the k^{th} goal, net after allowing for the available persons remaining in this category from the initial population.

The budget constraints are defined as

$$[\text{Cost vector in period } t]^T \; [\text{Total personnel vector in period } t] \leqslant B(t)$$

where

$$B(t) = \text{total dollar budget for personnel in period } t$$

and the T superscript stands for "transfers," as in (4). Substituting the expressions for the cost vector in period t and for the total personnel vector in period t we obtain

$$c^T(t) \; [M^t\bar{a} + \eta^1(t) + \eta^2(t)] \leqslant B(t)$$

where

$$c^T(t) \equiv [c_1(t), \ldots, c_n(t)].$$

Rearranging, we obtain the budget constraint

$$c^T(t) \; \eta^1(t) \leqslant B(t) - c^T(t) \; M^t\bar{a} - c^T(t) \; \eta^2(t).$$

Thus, in conclusion, the model (10) can now be replaced by

$$\min \sum_k \sum_t (E_k^+(t) + E_k^-(t))$$

subject to:

$$\begin{aligned} &\sum_{i \epsilon I_k} \eta_i^1(t) - E_k^+(t) + E_k^-(t) = \hat{g}_k(t) \\ &c^T(t) \; \eta^1(t) \leqslant B(t) - c^T(t) \; M^t\bar{a} - c^T(t) \; \eta^2(t) \\ &\eta^1(t) \geqslant M \; \eta^1(t\text{-}1) \\ &E_k^-, E_k^+ \geqslant 0. \end{aligned} \tag{30}$$

Should one wish to go still further, e.g., to also permit the transition matrix to change from period to period, all that would change in the above expressions would be to replace M^t by $M\begin{pmatrix} t\text{-}1 \\ t \end{pmatrix} M\begin{pmatrix} t\text{-}2 \\ t\text{-}1 \end{pmatrix} \ldots M\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and M by $M\begin{pmatrix} t\text{-}1 \\ t \end{pmatrix}$. See (14) ff.

APPENDIX AND PROTOTYPE EXAMPLE

As part of the initial report on the OCMM goal programming model for manpower planning (4), a purely hypothetical numerical illustration was described. Subsequently, it was demonstrated (6) that this hypothetical illustration could easily be solved using commercially available linear programming software. Testing of this formulation of the goal programming model has been started using "live" data. The data used in this initial test were obtained from the Personnel Automated Data System (PADS) of the U. S. Navy Office of Civilian Manpower Management and from estimates using source data obtained from the Office of Navy Comptroller Financial Resume of Civilian Employment Expenditures Fiscal Year 1968.

It should be remembered that the reports produced as a result of this test are of an experimental nature embodying assumptions which may not completely fit the specific situation being modelled. The main thrust of this test is to demonstrate that the OCMM goal programming model can utilize existing operational data files. Further, the test points to those areas where additional computer programs would be needed to implement the model on a prototype basis. Finally, and in the long run most important, the test begins to show how the model can be used to assist managers to make strategic manpower planning decisions.

In order to obtain manpower goals for this test, a prototype computer system designed to produce net civilian manpower requirements data was used.[1] This system, which uses the PADS data base, relates a manually-produced forecast of gross requirements by job type to projected numbers remaining of the base period population including transfers between job types. This allows the computation of accumulated net requirements[2] by job type by period. These computations did not explicitly include the retirement adjustments described in the paper but lumped the retirement data as part of the overall exit rate. Computer programs are in development to include explicitly the retirements feature. The salary and budgetary data used in the test were manually computed. Then, the test data were manually entered into the input format required by the linear programming software.

The job types used in this test are a combination of occupation and grade groupings. The U. S. Civil Service Commission occupation codes used include:

GS-201	Personnel Administration (General)
GS-212	Personnel Staffing
GS-221	Position Classification
GS-230	Employee-Management Relations
GS-235	Employee Development

The individual General Schedule grades have been aggregated to allow some scope for career planning without becoming excessively involved in planning for every grade in future years. This aggregation also simplifies the computer processing requirements. The grade groupings include:

GS 5-8	Trainee Level
GS 9-12	Middle Level
GS 13-14	Senior Level
GS 15-18	Executive Level

The transition probabilities and the related Markoff matrix, M, which we used in this test are arranged from ℓ to i.[3] Using blank cells to represent $M_{i\ell} = 0$, we obtained M as given in Table 1. These data were obtained by using a computer program to compare the PADS data files at two periods in time.[4]

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[1]For a detailed description of this computer system see A. Charnes, W. W. Cooper, R. J. Niehaus, and W. N. Price, "Application of Computer-Assisted Techniques to Manpower Planning," *The Journal of Navy Civilian Manpower Management* (forthcoming Fall 1969).

[2]The accumulated net requirements are the manpower goals or $g_k(t)$ contained in equation (1) of the basic paper.

[3]I.e., the $M_{i\ell}$ represents the transition rates from ℓ to i in each cell of M.

[4]See (7).

TABLE 1

PROFESSIONAL PERSONNEL OCCUPATIONS TRANSITION PROBABILITIES
NAVY-WIDE

(Transition Probabilities Arranged from ℓ to i)

i \ ℓ		201				212			221			230			235		
		5-8	9-12	13-14	15-18	5-8	9-12	13-14	5-8	9-12	13-14	5-8	9-12	13-14	5-8	9-12	13-14
201	5-8	.52				.07									.03		
	9-12	.15	.74			.01	.04			.02	.02	.03	.01		.03	.02	
	13-14		.06	.87	.02		.01	.07		.01	.07		.01				.02
	15-18			.02	.88												
212	5-8	.01				.41						.02					
	9-12	.03	.03			.28	.83	.02		.01			.01				
	13-14			.01			.01	.84									
221	5-8					.01			.38						.03		
	9-12	.01	.02				.01		.48	.78							
	13-14									.01	.84						
230	5-8	.01										.53					
	9-12	.02	.01			.01	.01			.01		.23	.83				
	13-14			.01									.03	.88			
235	5-8											.02			.43		
	9-12		.01						.02				.01		.29	.83	.02
	13-14															.02	.75

Experimental Report

The test was restricted to three periods. Table 2 contains the initial a_i values as components for the vector a of personnel already on board at the base period. Table 2 also contains the accumulated net manpower requirements or manpower goals. These were computed by subtracting from gross requirements the projected numbers of the base period population remaining. Gross requirements data were obtained by assuming that the requirements would essentially remain the same as the actual on board as of October 30, 1968. In some cases the gross requirements were adjusted upward to ensure that all manpower goals would be either zero or a positive value. In future tests this adjustment will be relaxed.

The salary values used in the test were obtained by averaging the fourth pay step for the grades represented in each grade grouping. These data were obtained from the Classification Act Salary Table in use for Fiscal Year 1969 and proposed for Fiscal Years 1970 and 1971.[1] The salary values in units of $1000 are:

Fiscal Year \ Grade Group	2 GS 5-8	3 GS 9-12	4 GS 13-14	5 GS 15-18
1969	6.8	10.7	16.1	21.0
1970	7.6	12.5	18.9	25.0
1971	7.6	12.5	18.9	25.0

The net civilian compensation budgets were obtained using an average salary value multiplied by total accumulated net requirements by period rather than through direct computations.[2] An average salary figure of $8460 was assumed for Fiscal Year 1969 using the figure given in the Office of Navy Comptroller Financial Resume of Civilian Employment Expenditures Fiscal Year 1968. This average salary figure was increased by 9.1% to $9250 for Fiscal Years 1970 and 1971 to account for the Federal pay raise. The net civilian compensation budgets used in units of $1000 were:

Fiscal Year	Total Accumulated Net Requirements	Projected Net Compensation Budget
1969	309	$2,620
1970	504	4,662
1971	710	6,568

The solution obtained using commercially available linear programming software is given in Table 3. Also included in Table 3 is a computation to show the recruiting requirements for each period.[3] Using this initial solution, parametric or alternative solutions can be obtained by changing the elements of the input data singly or in combination as desired. In this way the manpower manager can explore different combinations of manpower and financial information to shape the policy decisions required to accomplish the task under study.

[1]For fiscal years 1970 and 1971 the salary table reflects the Federal pay increase passed by Congress.

[2]As indicated by equation (4) of the basic paper, these budgets can be represented by $B(t) - c^T(t)\, M^t a$, where B(t) is the budgetary ceiling for civilian manpower expenditures in period (t), $c^T(t)$ is the vector of salary rates in period t and $M^t a$ is the vector of projected numbers of the original population remaining aboard in period t.

[3]The recruiting requirements were obtained by using equation (13) of the basic paper—$\xi(t) = \eta(t) - M\eta(t-1)$. This equation expresses the fact that the recruiting requirements for period t equals the net requirement for period t less the number remaining from period t-1.

TABLE 2

FORECASTED REQUIREMENTS PROFESSIONAL PERSONNEL OCCUPATIONS
NAVY-WIDE

Job Type	No. June 30, 1968	FISCAL YEAR 1969			FISCAL YEAR 1970			FISCAL YEAR 1971		
		Tot. Req.*	Proj. Orig. Pop. Left	Cum. Net Req.	Tot. Req.	Proj. Orig. Pop. Left	Cum. Net Req.	Tot. Req.	Proj. Orig. Pop. Left	Cum. Net Req.
201 GS 5-8	176	157	108	49	157	74	83	157	46	111
201 GS 9-12	447	474	410	64	474	371	103	474	328	146
201 GS 13-14	236	253†	253	0	263†	263	0	270†	270	0
201 GS 15-18	56	61	53	8	61	50	11	61	48	13
212 GS 5-8	116	100	56	44	100	40	60	100	25	75
212 GS 9-12	386	405	387	18	405	370	35	405	342	63
212 GS 13-14	48	49	48	1	49	48	1	49	47	2
221 GS 5-8	65	62	30	32	62	20	42	62	11	51
221 GS 9-12	286	295	267	28	295	236	59	295	203	92
221 GS 13-14	46	50	44	6	50	42	8	50	39	11
230 GS 5-8	58	45	34	11	45	23	22	45	15	30
230 GS 9-12	233	243	231	12	243	221	22	243	209	34
230 GS 13-14	48	52	51	1	55†	55	0	58†	58	0
235 GS 5-8	45	38	24	14	38	17	21	38	10	28
235 GS 9-12	247	252	238	14	252	225	27	252	208	44
235 GS 13-14	46	51	44	7	51	41	10	51	41	10
TOTALS	2,539	2,587	2,278	309	2,600	2,096	504	2,610	1,900	710

*Total requirements obtained by assuming that requirements would remain the same as the actual on board as of October 30, 1968.
†Adjusted to insure no negative accumulated net requirements. This adjustment to be relaxed in subsequent runs.

TABLE 3

PROFESSIONAL PERSONNEL OCCUPATIONS
NAVY-WIDE
ANALYSIS OF REQUIREMENTS USING LINEAR PROGRAMMING

Job Type	Fiscal Year 1969			Fiscal Year 1970			Fiscal Year 1971		
	Cum. Net From LP	Excess/Deficit From Goal*	Recommended New Hires For Period	Cum. Net From LP	Excess/Deficit From Goal	Recommended New Hires For Period	Cum. Net From LP	Excess/Deficit From Goal	Recommended New Hires For Period
201 GS 5-8	49		49	83		54	111		64
201 GS 9-12	64		64	74	-29	16	145	-1	73
201 GS 13-14	0		0	4	+4	0	10	+10	1
201 GS 15-18	0	-8	0	0	-11	0	0	-13	0
212 GS 5-8	44		44	60		49	75		49
212 GS 9-12	18		18	35		4	63		12
212 GS 13-14	0	-1	0	0	-1	0	1	-1	0
221 GS 5-8	32		32	13	-29	0	51		45
221 GS 9-12	28		28	59		20	55	-37	0
221 GS 13-14	0	-6	0	0	-8	0	1	-10	0
230 GS 5-8	11		11	22		18	30		18
230 GS 9-12	12		12	22		7	28	-6	1
230 GS 13-14	1		1	0		-1	1	+1	0
235 GS 5-8	14		14	21		15	28		13
235 GS 9-12	14		14	27		10	30	-14	1
235 GS 13-14	7		7	6	-4	0	5	-5	0
TOTALS	294	-15	294	426	-78	192	634	-76	277

*Goals are cumulative net requirements given in Table 2.

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A GENERALIZED NETWOF': MODEL FOR TRAINING AND RECRUITING DECISIONS IN MANPOWER PLANNING

by

A. Charnes
W. W. Cooper
R. J. Niehaus

Abstract

Models for manpower planning previously devised for the U. S. Navy's Office of Civilian Manpower Management have all utilized goal programming constructs with embedded Markoff processes. These models—referred to as "OCMM Mod«ls"—are here extended to include training elements along with related constraints.

Acknowledgment is due Wlodzinierz Szwarc, Carnegie-Mellon University for a careful review which yielded many helpful suggestions. Presented at XVII International Conference of The Institute of Management Sciences, London, England, 1970. Published in *Manpower and Management Science*, D. J. Bartholomew and A. R. Smith, eds. (London, English Universities Press, 1971).

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1. Introduction

A "goal programming model"[1] with iterated Markov elements to allow explicit consideration of transitions, exits, retirements, etc., in manpower planning over a sequence of periods was first introduced in [1]. As noted in the introduction to [1], the initial model formed one part of a total research effort on the part of the U. S. Navy's Office of Civilian Manpower Management (OCMM). This model—which we shall hereafter call the "OCMM Model"—has since been elaborated in a variety of directions including explicit allowance for predicted retirements within certain age or service categories and allowance for dynamically varying Markovian elements from period to period.[2] Other parts of the research program enunciated in [1] for the OCMM Models involve relating the multiple goals (e.g., the enunciated manpower ceilings) to tasks that need to be performed as well as introducing training possibilities as alternatives to recruitment and job transfers in order to meet (as closely as possible) the specified goals.[3]

In this report we propose to develop a first analytical model which explicitly provides for training as well as outside recruitment and job transition possibilities. We further propose to do this in a way which provides access to a variety of techniques such as "parametric variation" and "duality evaluations"[4] in order to facilitate experimentation with manpower program possibilities. In this way we shall be able to bring the power of linear programming to bear in evaluating optimal tradeoff possibilities and their resulting manpower mix and planning consequences. We shall then also be able to coordinate "career management" and "manpower planning" (and other parts of personnel planning)[5] by allowing for possible variations in manpower mixes and tradeoff possibilities in recruitment, transfer and training. Naturally, we shall want to accomplish this in a context that also considers other constraints such as financial budgets, supply and recruitment limitations imposed by policy or the environment at various times and also considers, of course, various kinds of limitations on training facilities. All except the last of these constraints have, however, already been included for explicit treatment in one or more versions of the OCMM Models. Hence we, shall here relate these constraints to their predecessor developments in order to be able to utilize some of the results already secured.

2. Modeling Strategies

In a manpower planning context it is natural to want to consider modeling for training in a way that allows for the effects of training and the selection of trainees on the manpower mixes which will be available in the years that follow such training.[6] This will be done here in a manner that provides direct contact with the goal programming developments in prior OCMM Models. A convenient way to accomplish this is to posit that the effect of training may be represented by a *different* matrix of transition rates which will then apply to personnel who have been selected for training. Then we can split the population into two groups—*viz.*, those selected for training in a specified period and those who are not selected for training. The latter group may than transit in accordnace with a "training" matrix while the former transit in accordance with "manpower" matrices of the kind we have previously utilized.[7]

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[1] See [1], Appendix B and Chapter X for an explanation and development of the ideas of goal programming.

[2] See, e.g., [5].

[3] Actually some of this has already been done in an earlier phase of this research program—see [8]—and in a way that takes account of on-the-job learning and dynamic organization design (and redesign) to allow for accumulating experience as well as formal training, etc.

[4] See [1], Chapters I and VI, for further explanations of these terms.

[5] *Vide* [9].

[6] Such effects should, of course, be interpreted as probabilistic projections when Markoff processes are being utilized in planning for a sequence of periods.

[7] See [3] and [5] where similar devices were used to obtain refinements for retirement and related considerations. The latter developments, including extensions to dynamically varying Markoff elements are, of course, also available for use in these OCMM Models (as noted in our introduction).

With the wanted contacts with previous OCMM Models thus established it is not necessary to treat again all of the previously developed types of constraints and stipulations and their related possibilities of variation and evaluation. Facilities as well as funds limitations on training capacities must be considered, of course, for training possibilities in each period. To continue to relate this to the previous developments, however, we shall here formulate only the funds limitation for explicit consideration. Situations where *no* facilities are available can then be modeled from a budgetary standpoint as having zero dollars available for that type of training.[1] In an analogous manner, period-by-period cost constraints may also be imposed for outside recruitment and, of course, we may then also continue to impose an overall budgetary constraint on salaries, and extend this part of the previous OCMM Models to include costs for recruiting and training as well as salary costs, etc., to be considered in each period, as desired.

In this same spirit we shall also preserve contact with the previous goal programming developments in our formulation of this model's objective. Thus we shall specify one part of this objective in terms of meeting a stipulated collection of manpower planning goals "as closely as possible," while staying within the constraints specified for each period in the planning horizon. We shall also extend the previously utilized objectives by including additional elements directed toward minimizing the total costs of outside recruitment and internal training. This extension is of interest in its own right, of course, but it also has the additional advantage of providing further insight for evaluating the relative weights assigned to deviations from other goals along with simultaneous evaluations of training-recruitment costs and tradeoffs.[2]

Finally we shall also want to utilize the types of transformations and reductions available from our prior research in order to develop special structures which would otherwise remain latent and, perhaps, unutilized for the computational advantages that such special structures can supply.[3] Indeed, the formulations we shall employ will give rise to a model structure which further generalizes the "network type" model relations that have been elaborated in [1].[4] In fact, we may regard the developments which we shall employ here as representing still *further* generalizations of the generalized network models presented in [1] but, to avoid a proliferation of terminology, we shall refer to this model, too, as a "generalized network type model."

3. Definitions and Development of Generalized Network Type

The above modeling strategy may be given analytical form as follows. Let[5]

$$\begin{aligned} x_{ij}(t) &= \text{number of personnel assigned to "job type } i\text{" from "source } j\text{"} \\ &\quad \textit{without} \text{ assignment to additional training in period } t \\ y_{ij}(t) &= \text{number of personnel assigned to "job type } i\text{" from "source } j\text{"} \\ &\quad \textit{with} \text{ assignment to training in period } t \end{aligned} \tag{1.1}$$

so that

$$x_{ij}(t) + y_{ij}(t) = \text{total number of personnel to be assigned to job type } i \text{ from source } j \text{ in period } t. \tag{1.2}$$

Thus, as indicated in the preceding section, the number of personnel obtained from source j may be summed and further distinguished between those assigned directly to job type i and those assigned to training for job type i. This will be done for each of the periods $t = 0, 1, 2, \ldots, N$ comprehended in the horizon for which manpower planning is being undertaken.

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[1]Alternatively, a lack of facilities can also be reflected in the training transition matrix by introducing zero rates of transition into certain parts of matrix and various combinations of budget and transition rate possibility may also be employed, of course.
[2]Of course, total salary and related cost considerations will also be available for evaluation, too, via the budgetary constraints.
[3]See, e.g., the discussion of "model types" in [1] Chapter 1 and ff.
[4]*Vide*, e.g., Chapter XVII in [1].
[5]As explained in [2], the terms "job type" and "source" are intended to comprehend distinctions between claimants or activities (for the same job or position) and recruitment or assignment in different geographical regions—and possibly other characteristics, too, if desired.

At time $t = 0$, the number of persons already in job type i may be represented by a known constant

$$a_i = \text{number of personnel in job type i within the organization at } t = 0. \tag{2}$$

Furthermore, to simplify notation, we may consider a_i as one of the components of a vector "a" comprehending all pertinent job types and, correspondingly, let

$$x^j(t) = \begin{bmatrix} x_{ij}(t) \\ \vdots \\ x_{nj}(t) \end{bmatrix}, \quad y^j(t) = \begin{bmatrix} y_{ij}(t) \\ \vdots \\ y_{nj}(t) \end{bmatrix} \tag{3}$$

represent vectors with n components for each of the job types $i = 1, \ldots, n$.

In proceeding toward our model objectives we shall want to allow for transfers between job types and also for the possibility that persons recruited for training in one category may subsequently transit to some other category. Thus, in keeping with previous developments, we introduce the Markoff matrix,[1] M, with elements

$$M_{i\ell} = \text{proportion of those in job } \ell\text{, without training in job } \ell\text{, who will transit to job i} \tag{4}$$

Then we introduce another Matrix, T, with elements

$$T_{i\ell} = \text{proportion of those in job } \ell \text{ with training in job } \ell \text{ who will transit to job i} \tag{5}$$

In order to bring the desired type of generalized network relations into prominence, we proceed as follows. For any period t, we can let

$x(t)$ = vector of personnel within the organization who are *not being* trained.[2]

$y(t)$ = vector of personnel within the organization who are being trained. (6.1)

$z(t)$ = vector of personnel from *outside* the organization who are being brought in.

Then we introduce the following types of relations,

$$z(1) + Mx(0) + Ty(0) = x(1) + y(1) \tag{6.2}$$

wherein $z(1)$ represents the vector of personnel recruited from outside while $Mx(0) + Ty(0)$ represents the transfer via jobs and training from inside and the whole splits into the two new vectors $x(1)$ for personnel not being trained and $y(1)$ for personnel who are being trained in period 1. The sum $x(1) + y(1)$ then represents the total number employed in each job type in period 1.

[1]Actually we will modify the usual Markoff representation and, just as we have done before, we omit one row and column to allow for the fact that in these OCMM Models entrance into the system is to be determined by reference to decisions on recruitment. Note, unlike other manpower planning and analyses which use Markoff processes, the decision variables and objectives are set forth explicitly along with other constraints, including policy limitations, etc.

[2]See [8] for developments in which job type assignments are also effected for their training potential on *other* types of jobs at subsequent times.

An evident extension of these developments now produces

$$\begin{aligned} a &= x(0) + y(0) \\ 0 &= -Mx(0) - Ty(0) + x(1) + y(1) \quad -z(1) \\ 0 &= \qquad\qquad -Mx(1) - Ty(1) + x(2) + y(2) \quad -z(2) \end{aligned} \tag{7.1}$$

and so on via

$$\begin{aligned} 0 &= -Mx(t-1) - Ty(t-1) + y(t) \quad -z(t) \\ 0 &= \qquad\qquad -Mx(t) - Ty(t) + x(t+1) + y(t+1) - z(t+1) \end{aligned} \tag{7.2}$$

where $z(0) \equiv 0$.[1] Reference to this structure suggests incidence relations of the generalized network variety. Of course, the indicated incidences are on vectors x(t), y(t) and matrices M and T but the representations (7.1) and (7.2) nevertheless display a structure which lends itself to this symbolism and related interpretations as a further generalization of these generalized network concepts and developments which we have previously used to advantage for serving computational efficiencies. See [1] Chapter XVII ff.

4. Additional Constraints and Objectives

Other constraints will also be needed, however, to allow for limitations on training and recruitment. As indicated in preceding sections, we want to relate these to previously utilized formulations of budgetary limitations in OCMM Models. Thus, we continue from (6.1) and introduce the following "training constraints"

$$K^1(t)\, y(t) \leqq d^1(t) \tag{8}$$

where $K^1(t)$ is a matrix of the costs for training for each job type and source at time t and $d^1(t)$ is a stipulated vector of limitations imposed on such expenditures.

In this same vein we may also represent the constraints on outside recruitment via

$$K^2(t)\, z(t) \leqq d^2(t) \tag{9}$$

where $K^2(t)$ is a matrix of recruitment costs at time t and $d^2(t)$ is a corresponding vector of stipulations.

Finally we insert budget constraints on total salaries, as in previous OCMM Models,

$$c^T(t)\ [x(t) + y(t)] \leqq B(t)$$

where

$$\begin{aligned} c^T(t) &= \text{transpose of the vector of salaries to be paid for each job type in period } t \\ B(t) &= \text{budget limitation (a scalar) on total salaries which may be paid in period } t. \end{aligned} \tag{10}$$

The above constraints may be adjoined to those exhibited in the preceding section. Then letting

$$\begin{aligned} f_k(t) &= \text{prescribed ceiling for } k^{th} \text{ type of manpower.} \\ \mu_{kt} &= \text{weight assigned to deviation from } k^{th} \text{ manpower ceiling in period } t \end{aligned} \tag{11}$$

[1] Evidently we can also replace M and T by time-dependent Markoff matrices as in [5] and [8].

we can formulate the objective for this model as

$$\min \sum_{t=1}^{N} \sum_{k} \mu_{kt} \left| e_{I_k}^T \, [x(t) + y(t)] \; -f_k(t) \right| + \sum_{t=1}^{N} \mu_{ot} \, [\alpha^T y(t) + \beta^T z(t)] \tag{12}$$

where α^T and β^T represent vectors (transposed) containing the recruiting and training costs elements to be considered in the objective while

$$e_{I_k}^T \equiv \begin{pmatrix} 0 & , & 1 \\ ixI_k & & ixI_k \end{pmatrix} \tag{13}$$

is a vector which has zeros in all components except those which are the unity elements. The latter, i.e., the values of unity, are in the positions that correspond to the k^{th} type of manpower.

5. Matrix and Structure

Drawing the elements of the preceding two sections together we may obtain the matrix represented in Table 1. This may be used for developing direct or dual relations if desired but here we have only utilized the direct variable x(t), y(t) and z(t) as defined and developed in the preceding section.[1] Such extensions will require replacing the (nonlinear) absolute value terms in (12) by their linear programming equivalent via the usual "goal programming" reductions.[2] This has been done in the preceding papers in this series, however, and hence need not be repeated here.

With this structure now being available the stage is set for further interpretation and extensions. This will be done in a supplemental report, however, and made more concrete by means of a simple numerical example and just as was done for the reports finally incorporated in [5] this will be accompanied by related computations and solution results. The portrayal in Figure 1, for the time being, then completes the present report.

6. Addendum: Some Extensions and Interpretations

This paper was to conclude with numerical examples to illustrate the above developments. When presented in one of the Manpower Planning Sessions at the TIMS meetings for the XVII International Conference,[3] however, some of the comments and questions indicated that another course might be preferable. We had been relying on only general discussion and the underlying mathematical development to indicate how further elaborations and interpretations might be essayed from this series of OCMM Models. This appears to have been inadequate and so it may be desirable to make at least some of these possibilities explicit at the present juncture.

First we observe that this model involves embedded Markoff processes and hence is probabilistic in character. The interpretations need to be arranged accordingly and so do some of the further extensions that are possible. To detail all of these is beyond the scope of the present paper (as well as beyond the immediate applications possibilities that guided the developments to this point). We shall proceed only via a very simple example and develop it only to a degree that will help to suggest some of these further possibilities.

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[1] These developments implicitly utilize transformations and reductions first introduced in [5] for simplifying matters and making the underlying structure apparent as a guide to computational and interpretative developments.

[2] See Chapter X in [1] for a general development including geometric interpretations and analytical developments of the theory underlying these reductions.

[3] At the Imperial College in London, England, on July 2, 1970.

DESCRIPTION	STIPULATIONS	x(0)	y(0)	x(1)	y(1)	x(2)	y(2)	...	x(N-1)	y(N-1)	x(N)	y(N)	z(1)	z(2)	...	z(N)
NETWORK TRANSITIONS																
	a	I	I													
	0	-M	-T	I	I								-I			
	0			-M	-T	I	I							-I		
	...		.	. .	. .	. .	. .		. .	. .	. .	. .	.	. . . . .		. . .
	0								-M	-T	I	I				-I
TRAINING CONSTRAINTS	>															
	$d^1(1)$				$K^1(1)$											
	$d^1(2)$						$K^1(2)$									
	...		.	.	. .	.	. . .		. . .	. . . .			. . . . .	. . . . .		. . . .
	$d^1(N)$											$K^1(N)$				
OUTSIDE RECRUITMENT	>															
	$d^2(1)$												$K^2(1)$			
	$d^2(2)$													$K^2(2)$		
	...		.	.	. .	. .			. . .	. . .	. . .		. . . .	. .		
	$d^2(N)$															$K^2(N)$
SALARY BUDGET	>															
	B(1)			$c^T(1)$	$c^T(1)$											
	B(2)					$c^T(2)$	$c^T(2)$				.					
	...		.	. . .		. .	.		. . . . .	. . . . .			. . .			
	B(N)										$c^T(N)$	$c^T(N)$				

NOTE $x(t), y(t), z(t) > 0$, all t.

Figure 1. MANPOWER PLANNING RELATIONS

For this development we revert to the simpler cases of the earlier OCMM Models—see, e.g., [2] and [4]—in which no training transitions are included for explicit consideration. We also restrict the example to one involving only one source and two job types with matrix

$$M = \begin{bmatrix} .7 & .1 \\ .2 & .6 \end{bmatrix} \tag{14}$$

This matrix details the probabilities of the transitions which may be effected *from* the column, $\ell = 1, 2$, *to* the row, $i = 1, 2$, for the job types associated with these indices. It may be thought of as a matrix of "Markoff type" adjusted for the fact that we want to use it as part of a goal programming model in which new entrants into the system are effected via the decision variables $x_{ij}(t)$—wherein the values assigned to each such x denote the number of recruits from source j for job type i in each period t. See (1) ff. To accommodate the wanted variation from ordinary uses and developments in Markoff analyses, we have introduced the convention of omitting the column of the matrix that is usually allotted for the new entrant probabilities. We have also omitted the row for the exiting probabilities[1] and thereby obtained a characterization in which the matrix, -M, is square.

In the simple case being considered here, there are only two job types and one source being considered for the decision variables $x_{ij}(t)$. Hence, we can eliminate the index j (which is needed only when there is more than one source that must be identified) and write,

$$X(1) = \begin{bmatrix} x_1(1) \\ x_2(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{15}$$

when a plan calls for hiring one person for job type 1 and none for job type 2 in period 1. We distinguish these planning decisions from the corresponding variates by writing the latter as

$$X(1) = \begin{bmatrix} \hat{x}_1(1) \\ \hat{x}_2(1) \end{bmatrix} \tag{16}$$

with values to be determined via random processes generated from (14).

To clarify this last statement we commence with the initial state vector

$$a^T = (a_1, a_2) = (1, 1) \tag{17}$$

and then restrict our immediate attention to discrete state possibilities which can be generated from (17) via (14). These states and their corresponding probabilities are obtained from (14) and (15). For instance, for job type 1, we obtain

$$P\left[\hat{x}_1(1) = 0/a_1 = 1, a_2 = 1\right] = q_{11}\, q_{12} \equiv (1 - p_{11})(1 - p_{12}) = .27$$

as the probability of this event.[2] This and the other probabilities are thus evaluated via

$$\begin{aligned} P\left[\hat{x}_1(1) = 0 \mid a_1 = 1, a_2 = 1\right] &= (1 - .7) \times (1 - .1) &&= .27 \\ P\left[\hat{x}_1(1) = 1 \mid a_1 = 1, a_2 = 1\right] &= .7 \times (1 - .1) + (1 - .7) \times .1 &&= .66 \\ P\left[\hat{x}_1(1) = 2 \mid a_1 = 1, a_2 = 1\right] &= .7 \times .1 &&= .07 \\ & \text{Total} && \underline{\underline{1.00}} \end{aligned} \tag{18}$$

[1]These may always be retrieved, if wanted, as the difference between unity and the corresponding column sum in M.
[2]See Appendix.

Applying these probabilities to the corresponding states produces

$$E\,\hat{x}_1\,(1) = .27 \times 0 + .66 \times 1 + .07 \times 2 = .80 \tag{19.1}$$

for the expected value. Alternatively, the straightforward matrix computation,

$$Ma = \begin{bmatrix} .7 & .1 \\ .2 & .6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} .8 \\ .8 \end{bmatrix} \tag{19.2}$$

gives the same expected value for job type 1 (and job type 2) for period 1.

This is, of course, the expected value that results only from the initial state vector for the job types given in (17). The planned values for the decision variables, however, can also be aligned with this kind of probabilistic development via

$$M^o\,X\,(1) + M^1 a = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} .8 \\ .8 \end{bmatrix} = \begin{bmatrix} 1.8 \\ 0.8 \end{bmatrix} \tag{20}$$

where we have used the X (1) values given in (15). Here $M^o \equiv I$, the identity matrix. The latter can be accorded the interpretation of a Markoff matrix, too, in which the transition probabilities are unity on the diagonal and zero elsewhere.[1] This means that there is unit probability of remaining in the job type for which the corresponding personnel components of X (1) were recruited during period 1. Under this interpretation an expected value is also obtained which may be added to those secured via the transitions effected from the components of a in period 0. The resulting sums provide the expected number of occupants for each job type that the plan will provide for period 1. Expression (20) can then be interpreted accordingly.

Of course, other probabilistic aspects may also need to be considered and a development that will suggest some of these possibilities may also be synthesized from this same very simple example.

From the computations in (17) we may, for instance, observe that

$$P\left[\hat{x}_1(1) \geqslant 1 \mid a_1 = 1, a_2 = 1\right] = .73. \tag{21}$$

If this is not satisfactory then either the initial components in the vector, a, must be altered or else the matrix of transition probabilities must be adjusted[2]—or, possibly, both of these might be done in various combinations.[3] For example, if we altered (17) to

$$a^T = (a_1, a_2) = (2, 1) \tag{22}$$

we would obtain

$$\begin{aligned}
P\left[\hat{x}_1\,(1) = 0 \mid a_1 = 2, a_2 = 1\right] &= .3 \times .3 \times .9 &&= .081 \\
P\left[\hat{x}_1\,(1) = 1 \mid a_1 = 2, a_2 = 1\right] &= 2(.7 \times .3 \times .9) + .3 \times .3 \times .1 &&= .387 \\
P\left[\hat{x}_1\,(1) = 2 \mid a_1 = 2, a_2 = 1\right] &= .7 \times .7 \times .9 + 2(.7 \times .3 \times .1) &&= .483 \\
P\left[\hat{x}_1\,(1) = 3 \mid a_1 = 2, a_2 = 1\right] &= .7 \times .7 \times .1 &&= .049 \\
&\text{Total} && \underline{\underline{1.000}}
\end{aligned} \tag{23}$$

[1]These are so-called "trapped state" probabilities.

[2]Perhaps by behavioral science research designed to ascertain how the relative attractiveness of these job types may be altered to produce higher and lower transition probability values in selected categories. (Note that in this view a guide for such behavioral science research is thereby automatically supplied for simultaneous consideration of career management and manpower planning.)

[3]Depending on their relative costs and the further benefits that might attend such alteration. Notice, for instance, that the use of these models for manpower planning can also affect the psychological attitudes and job-preparation propensities that personnel may subsequently display and these possibilities, too, need to be considered in a comprehensive program of personnel research.

This gives

$$P\left[\hat{x}_1\ (1) \geqslant 1 \mid a_2 = 2, a_2 = 1\right] = .919 \tag{24}$$

and also

$$E\ \hat{x}_1\ (1) = .081 \times 0 + .387 \times 1 + .483 \times 2 + .049 \times 3 = 1.5. \tag{25}$$

Compared with (19.1) and (21) the alteration in initial conditions when proceeding from (17) to (22) increases the expected value and the probability for this job type, as exhibited in (25) and (24).

Of course, the expected cost of the latter result will also be increased, in general, and this, too, needs to be considered by the model–over the entire horizon. For instance, this increase may make it possible to reduce the value of $x_1(1) = 1$ in (15) with resulting benefits later on. The point to bear in mind here, in any event, is that the OCMM Models have been developed for use in a context such as the U. S. Department of Defense's Program-Planning and Budgeting Systems (PPBS), which envisions a 5-year plan–or forecast of such plans–tied into corresponding fiscal-monetary, and other, requirements. This, in turn, confines these models to a use of only "zero order decision rules"–to use the terminology of "chance constrained programming,"[1] but, of course, other applications may admit of access to other classes of decision rules[2] as well.

To state all of this in a different manner, the distinctions we have elsewhere introduced for dealing with "planning" as contrasted with "control" and "operating" aspects of management are here omitted.[3] That is, it is assumed that the controls are perfect and that what is wanted is a detailing of manpower plans and related decisions over some specified future time horizon. This does not mean that possibilities for subsequent modification of these plans are eliminated. It also does not mean that the fact that such plans are formed in a probabilistic context is ignored. It only means that the relevant planning decisions are all to be specified numerically–including even those parts which are used to control the underlying probabilities and their related risks–so that, e.g., they may be coordinated[4] with the other plans and considerations that are being obtained, perhaps from other parts of the Department of Defense organizations.

The significance of these characterizations may be clarified, perhaps, by extending the immediately preceding developments. We shall do this by showing how such probabilistic considerations may be dealt with via values assigned to the decision variables (in earlier periods), but without entering into the complex chains of conditional probability computations that can attend these extensions in their many varieties.

Notice, for instance, that the alteration form (17) and (22) has produced a change in the probability distribution for the period 1 states. Similarly, the X (1) choices can also produce different probability distributions for the possible states in period 2–as well as in the combinations for compounding still other probability distributions in subsequent periods. This suggests that, in a manner analogous to the DEMON models,[5] constructs of chance-constrained programming can also be brought to bear in order to "constrain out" any of these distributions that are considered to be undesirable–e.g., with reference to the risk properties associated with their tails. For instance, via expressions of the form

$$P\left[\hat{x}_1\ (2) \geqslant k \mid a_1, a_2, x_1\ (1), x_2\ (1)\right] \geqslant \alpha \tag{26}$$

one may assure that k, a prescribed minimum number of "experienced personnel,"[6] will be available for assignment in job type 1 at the start of period 2. This minimum value is to be assumed with a specified

[1] See, e.g., [20].
[2] See, e.g., [24].
[3] See, e.g., Preface and Chapter 1 in [1].
[4] It must be remembered that budgets perform *coordination* as well as *control* and *planning* functions.
[5] The reference is to models developed in cooperation with J. K. DeVoe and D. B. Learner (then at Batten, Barton, Durstine & Osborn, Inc.) for use in marketing new products. See, e.g., [21] and [22].
[6] I.e., experienced in the organization, although not necessarily in this particular job type. (Some of these persons may have transited from the x_2 (1) choices in period 1 as well as the initial members of this second job type given by a_2.)

probability, $0 < \alpha < 1$. With a_1 and a_2 given this means that the choices of x_1 (1) and x_2 (1) must be made in a way that eliminates all of the probability distributions which do not satisfy these conditions. This can be done via expressions of the form

$$(M)_1^2\ a + (M)_1 \times (1) \geq h\ (\alpha, k) \tag{27}$$

in which $(A)_i$ refers to the i^{th} row of any matrix, A, while h depends on the α and k prescribed in (26). In principle, expressions such as (27) may be ascertained in advance[1] from the original probability data but, as a practical matter, recourse to approximation procedures and specially devised computer routines will need to be designed for these purposes. Similar controls may also be imposed on the distributions for $\hat{x}_2$ (2) and these may continue into subsequent periods, too, in a variety of ways.

The above types of developments are all comprehended in the plans for research at OCMM. Jumping still further ahead of the immediate applications possibilities, however, one may consider developments in which the matrix components–i.e., the transition probabilities–are also random variables, and even this does not end a trail that appears to stretch far beyond present practices and data limitations and which, at the same time, invites extensions in these directions. Turning back to matters of more immediate moment, however, it is perhaps of interest to observe that the elements in these Markoff matrices may be estimated via constrained regression procedures of the type for which goal programming was originally developed.[2] This also raises a series of research possibilities for exploring the properties of statistical estimators developed from data (e.g., OCMM plans) in which the observations are also generated from the same (e.g., goal programming) procedures.

Bearing these extensions and interpretations in mind, it seems best now to turn to other questions raised in the Sessions where this paper was first presented. These concerned the need for considering training and other possibilities that stretch across two (and possibly more) periods.

To answer these kinds of questions it is evidently necessary to return to the models in earlier sections of this paper. Thus we shall assume that training matrices, T, as well as job-transition matrices, M, are to be comprehended by models of the kind portrayed in (7.1) ff. Then we may observe that these T and M matrices may be varied by period, if desired, to accommodate different training and job transition probabilities.

Using $(T)_i$ for the i^{th} row of T in some time period, t-1, say, we may write

$$y_{ij}\ (t) = (T)_i\ y^j\ (t\text{-}1) \tag{28}$$

in order to constrain the period t candidates for "job type" (or "training type") i to those persons previously trained in period (t-1). Observe, however, that the application of row i from T permits some persons to be considered for training in the i^{th} category in period t even though they were not in this category during period t-1, provided there is some positive probability of this occurring in a cell corresponding to a non-zero recruitment for this component of the vector y (t-1).

A variety of such constraints may be employed. For instance, sums over the various sources j may be formed along with other restrictions and refinements to control the expected quantities (and the related statistical distributions) for consideration in the planned programs of training. Also, a proportion $0 \leq d \leq 1$ may be assigned for training in job type k by inserting additional constraints of the form

$$y_{kj}\ (t) = dy_{ij}\ (t) \tag{29}$$

with the remainder being assigned to training for job type i in period t.

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[1] We are not addressing considerations such as the existence of solutions, etc., which may arise when inequalities such as (26) or (27) are inserted into these goal programming models.

[2] It is worth observing, perhaps, that goal programming was originally developed in the context of an applied problem of personnel administration–i.e., the problem of determining a best salary structure under hierarchical organization constraints in order to meet competition with only partially known data. See, e.g., Chapter X in [1].

Evidently the above characterizations may be varied across periods and cumulatively, too, if desired. They may also be extended to job as well as training transitions as when, for instance, experience in one or more preceding jobs for a succession of periods may be established, perhaps probabilistically,[1] as a prerequisite for access to other jobs. Many such possibilities are present and may be tried and evaluated in the course of research on these kinds of models.

A variety of constraints, probabilistic or otherwise, can evidently be used to cross over or relate phenomenon in any or all periods of interest. This means, in turn, that the transitions from one state to another may be determined via probabilities which are far removed from the immediately adjacent times and states. In this respect, too, some of the rigidities and limitations of the ordinary one-state Markoff transitions may also be modified and ameliorated via the constraining relations which we are using to extend and adapt the classical types of Markoff analyses. Note, for instance, that we can employ constraints which relate present transition to future as well as past probabilities and thereby bring into play the "pull" of future job prospects as well as the "push" of past careers and experiences.

The above remarks may help to suggest some of these additional possibilities for research and also serve to clarify some of the developments that we have already been presenting from the research being undertaken at OCMM. Time and the vagaries of the current American scene permitting, further results from the OCMM research program will be released in future reports and conferences.

[1]Cf. the characterizations provided for distinguishing between "policies" and "rules" via chance constraints, as discussed in [23].

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APPENDIX

Let $p_{ij} \geqslant 0$, $i=1, \ldots, n$, represent elements in a "Markoff type" matrix and define $q_{ij} = 1-p_{ij} \geqslant 0$ for every i, j. Let x_j be the value (discrete) of the j^{th} component, $j=1, \ldots, n$, of any state vector and let $P(\hat{x}_i = k)$ represent the probability that $\hat{x}_i$, the i^{th} random variable generated from these x_i and p_{ij}, will assume the integer value $k = 0, 1, 2, \ldots, y$;

$$y \equiv \sum_{j=1}^{n} x_j.$$

Then

$$\sum_{j=1}^{n} p_{ij}\, x_j = \sum_{k=0}^{y} k\, P(\hat{x}_i = k) = E\, \hat{x}_i, \tag{1}$$

where E represents the expected value operator.

This theorem, which is utilized in the immediately preceding text, does not seem to be readily available in the literature that deals with Markoff processes.[1] We will therefore prove it here, after some preliminary definitions and developments.

In our case,[2]

$$P(\hat{x}_i = k) = \sum_{C_k^y} \prod_{j=1}^{n} C_{r_j}^{x_j}\, p_{ij}^{r_j}\, q_{ij}^{x_j-r_j} \tag{2}$$

where the indicated summation is over all of the C_k^y terms for which $r_1 + r_2 + \ldots + r_n = k$ and r_j is the number of x_j appearing in the latter sum and

$$C_{r_j}^{x_j} = \begin{cases} \dfrac{x_{j!}}{r_{j!}\,(x_j - r_j)_!}, & x_j \geqslant r_j \\ 0 & \text{, otherwise} \end{cases} \tag{3}$$

where $x_{j!} = x_j\,(x_j - 1)\,(x_j - 2) \ldots 1$. Also, "Π" represents "product" as in

$$\prod_{j=1}^{n} C_{r_j}^{x_j}\, p_{ij}^{r_j}\, q_{ij}^{x_j-r_j} = C_{r_1}^{x_1}\, p_{i1}^{r_1}\, q_{i1}^{x_1-r_1}\, C_{r_2}^{x_2}\, p_{i2}^{r_2}\, q_{i2}^{x_2-r_2} \;.\;.\; C_{r_n}^{x_n}\, p_{in}^{r_n}\, q_{in}^{x_n-r_n} \tag{4}$$

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[1] Possibly because a good deal of the classical literature in this area is based on natural science examples and models from physics and biology which are observation rather than decision oriented. (They also do not deal with constrained relations and their Markoff matrices have $\sum_j p_{ij} = 1$.)

[2] Extensions to continuous cases and higher moments are possible but will not be developed here.

Using the above definitions and developments our proof will now be developed in a way that relates it directly to the preceding text. We also want to develop it in a manner that naturally relates it to familiar uses of the moment generating functions for binomial distributions. We therefore commence with the case for which the Markoff-type matrix is 1x1 with a single element, p, and write[1]

$$(pt + q)^x = \sum_{k=0}^{x} t^k C_k^x p^k q^{x-k} \tag{5}$$

Differentiating with respect to t gives

$$xp(pt + q)^{x-1} \sum_{k=0}^{x} kt^{k-1} C_k^x p^k q^{x-k} \tag{6}$$

and setting t=1 then produces

$$xp = \sum_{k=0}^{x} k C_k^x p^k q^{x-k} = \sum_{k=0}^{x} k P(\hat{x}=k) \tag{7}$$

so that (1) evidently holds for the 1x1 Markoff type matrix.

To relate (1) specifically to the example in the preceding text we establish

$$\sum_{j=1}^{2} p_{ij} x_j \sum_{k=0}^{y} k P(\hat{x}_i=k); \; y \equiv x_1 + x_2, \tag{8}$$

where i=1, 2 in the 2x2 matrix with elements p_{ij} and $p_{ij} + q_{ij} = 1$–i, j=1, 2. For a proof in this case we extend the above development via the binomial moment generation functions,

$$\prod_{j=1}^{2} (p_{ij} t_j \qquad t_j^{r_j} C_{r_j}^{x_j} p_{ij}^{r_j} q_{ij}^{x_j-r_j}. \tag{9}$$

The total differential for this expression gives

$$x_1 p_{i1} (p_{i1} t_1 + q_{i1})^{x_1-1} (p_{i2} t_2 + q_{i2})^{x_2} dt_1 + x_2 p_{i2} (p_{i1} t_1 + q_{i1})^{x_1} (p_{i2} t_2 + q_{i2})^{x_2-1} dt_2 =$$

$$= \left[\sum_{r_1=0}^{x_1} r_1 t_1^{r_1-1} C_{r_1}^{x_1} p_{i1}^{r_1} q_{i1}^{x_1-r_1} \right] \left[\sum_{r_2=0}^{x_2} t_2^{r_2} C_{r_2}^{x_2} p_{i2}^{r_2} q_{i2}^{x_2-r_2} \right] dt_1 + \tag{10}$$

$$+ \left[\sum_{r_1=0}^{x_1} t_1^{r_1} C_{r_1}^{x_1} p_{i1}^{r_1} q_{i1}^{x_1-r_1} \right] \left[\sum_{r_2=0}^{x_2} r_2 t_2^{r_2-1} C_{r_2}^{x_2} p_{i2}^{r_2} q_{i2}^{x_2-r_2} \right] dt_2 .$$

This is an identity in the dt's. Hence we may equate coefficients, so that, at $t_1 = t_2 = 1$,

[1]We define $0^0 = 1$ so that, e.g., fully stochastic (ordinary) Markoff matrices are also included.

$$\sum_{j=1}^{2} p_{ij} x_j = \sum_{r_1=0}^{x_1} \sum_{r_2=0}^{x_2} (r_1 + r_2) C_{r_1}^{x_1} C_{r_2}^{x_2} p_{i1}^{r_1} q_{i1}^{x_1-r_1} q_{i2}^{x_2-r_2} p_{i2}^{r_2} =$$

$$= 0 C_0^{x_1} C_0^{x_2} p_{i1}^0 q_{i1}^{x_1} q_{i2}^{x_2} p_{i2}^0 + \ldots + k \sum_{s=0}^{k} C_{k-s}^{x_1} C_s^{x_2} p_{i1}^{k-s} q_{i1}^{x_1-(k-s)} q_{i2}^{x_2-p} p_{i2}^{p} + \qquad (12)$$

$$+ \ldots + (x_1 + x_2) C_{x_1}^{x_1} C_{x_2}^{x_2} p_{i1}^{x_1} q_{i1} q_{i2}^0 p_{i2}^{x_2} =$$

$$= \sum_{k=0}^{x_1+x_2} k \left[\sum_{s=0}^{k} C_{k-s}^{x_1} C_s^{x_2} p_{i1}^{k-s} q_{i1}^{x_1-(k-s)} q_{i2}^{x_2-s} p_{i2}^{s} \right]$$

or, in the notation of (1) and (2), above,

$$\sum_{j=1}^{2} p_{ij} x_j = \sum_{k=0}^{y} k P(\hat{x}_i=k) = \sum_{k=0}^{y} k \sum_{C_k^y} \prod_{j=1}^{2} C_{r_j}^{x_j} p_{ij}^{r_j} q_{ij}^{x_j-r_j} \qquad (12)$$

where, as indicated for (2), the second of the latter sums is to be considered over all of the C_k' terms for which $r_1 + r_2 = k$. The derivation leading to (12) then shows that the theorem is also true in this case for *either* i = 1, 2.

The general case can evidently be obtained by a direct extension of the above arguments . . . *viz.*,

$$\prod_{j=1}^{n} (p_{ij} t_j + q_{ij})^{x_j} = \prod_{j=1}^{n} \left[\sum_{r_j=0}^{x_j} t_j^{r_j} C_{r_j}^{x_j} p_{ij}^{r_j} q_{ij}^{x_j-r_j} \right].$$

Then differentiating and setting all t=1 produces

$$\sum_{j=1}^{n} p_{ij} x_j = \sum_{r_1=0}^{x_1} \sum_{r_2=0}^{x_2} \ldots \sum_{r_n=0}^{x_n} (r_1 + r_2 + \ldots + r_n) \prod_{j=1}^{n} C_{r_j}^{x_j} p_{ij}^{r_j} q_{ij}^{x_j-r_j} =$$

$$= 0 \prod_{j=1}^{n} C_0^{x_j} p_{ij}^0 q_{ij}^{x_j} + \ldots + (x_1 + x_2 + \ldots + x_n) \prod_{j=1}^{n} C_{x_j}^{x_j} p_{ij}^{x_j} q_{ij}^0$$

$$= \sum_{k=0}^{y} k \sum_{C_k^y} \prod_{j=1}^{n} C_{r_j}^{x_j} p_{ij}^{r_j} q_{ij}^{x_j-r_j} = \sum_{k=0}^{y} k P(\hat{x}_i=k).$$

Q.E.D.

MULTI-LEVEL MODELS FOR CAREER MANAGEMENT AND RESOURCE PLANNING

by

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Abstract

The OCMM models for manpower planning were initially formulated in terms of a goal-programming, Markoff process combination with the latter embedded in the former. This subsequently admitted of a further extension in which the Markovian elements were reformulated so that they could be submitted to chance-constrained programming use and interpretations. They made it possible to plan for personnel recruitment, training, retraining and stochastically determined transfers, so that selected constraint stipulations would be honored with prescribed probabilities ir any set of differing time intervals for which such constraints might be imposed. Here, a further extension is made: An input-output (Leontief) analysis of an often one-period variety is embedded in a goal programming formulation which is subsequently extended to a two-period dynamic formulation. Applications to manpower planning and resource allocation are examined for their support as well as their direct activity implications.

A further development via chance-constrained programming in a different direction is then provided which comprehends elements in the objective, as well as the constraints, in a formulation that can then be interpreted as an extension of goal programming to such probabilistic considerations.

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Introduction

This report discusses new and flexible techniques designed to assist in career management, resource planning and other (related) parts of manpower planning and personnel management. These techniques revolve around certain models and modelling strategies which were developed in the research program at the Office of Civilian Manpower Management (OCMM) of the U. S. Navy. This research program is aimed at providing new and improved methods for civilian manpower management and planning. In examining ways to assist in obtaining a better allocation of civilian manpower, what is found almost immediately is the fact that there are multiple objectives and multiple variables (e.g., types of support units, types of resources, time periods, etc.) which interact with one another. There are also cascading, dynamic effects which must be taken into account since decisions made in one period have consequences and hence affect the decisions for succeeding periods.

There is, very naturally, a desire to achieve reasonable balances between the input resources and the output goals in each period as well as over a range of periods. The present report is, in fact, directed to this desire in the light of the preceding considerations. We shall therefore try to outline certain approaches which can aid in tying all these considerations together while, at the same time, building on previous modelling work and existing data files. In particular, we shall move these developments in a direction which ties them to the work of others in areas like the use of input-output analysis in resource allocation decisions and chance-constrained programming approaches to risk control and evaluation. This will be accompanied by a series of numerical examples to facilitate reader understanding and evaluation and also to exhibit the ways in which the model structure depends upon information requirements and data availability.

An objective of the OCMM modelling research program is to provide one of the inputs in the development of operational manpower and career management information systems. Earlier research has already substantially influenced these information system developments. Some thirty-five special studies (mainly having to do with historical movement and retirement statistics) were supported using the computer software capability[1] developed as a result of the research. These included such studies as a Navy-wide assessment for the Assistant Secretary of the Navy (Installations and Logistics) of the career progression of the 120,000 blue collar employees in maintenance-related occupations. Also, the computer system is being modified to produce projected data on retirements to assist in the development and justification of Navy budget submissions. It is worth noting that these studies could not have been made without the extensive research into suitable job category coding systems and computer programming techniques necessary to support the mathematical research.

In addition to the supporting computer software mentioned above, considerable testing has been completed of the models themselves. Trial runs have been made of a number of examples with the largest containing 1500 constraints and 2500 variables.[2] All of this is mentioned to show the progress of earlier research. It also points to the fact that research into implementation techniques is as important as the basic mathematical research if one is to end with a usable product.

Modelling Strategy

Preceding work as discussed in (7)–(12) has opened a way for dealing simultaneously with manpower planning and career management. The models covered in this report are therefore also oriented in this direction.

[1] I.e., The Five Year Navy Civilian Manpower Plan (FYNCIMP) subsystem of the Navy Automated Civilian Manpower Information System (NACMIS).

[2] Currently, an example containing 3200 constraints and 5200 variables has been developed and is being tested.

Here, however, manpower planning is considered to be concerned with aggregate resource allocation while career management is considered to involve man-job matching either at the skill or individual job level.[1] Much of this has been done already, of course, in the context of earlier OCMM models. These OCMM models are of the type called goal programming, where the objective is to determine a program which meets "as closely as possible" a set of manpower goals in each period considering various constraints. The OCMM models were also subsequently extended to include training requirements and job requirements and thus provide a further aid in career planning.[2] While keeping these past developments in view, however, the present paper will proceed to some further extensions which will also help to make contact with the original work of others in areas like input-output analysis, etc.

Some further background is now perhaps in order. It is felt to be important to bring these models into contact with the program planning budgeting or PPB systems in being or under development within the Navy. This is important because such planning actually drives the manpower allocation and career management process. Since the development of PPB systems beyond the methodology of allocating civilian manpower is not the direct responsibility of OCMM, an investigation was made of current and proposed methodologies in the Office of the Secretary of Defense as well as within the Navy. This naturally led us to consider the models under development in the Center for Naval Analyses for the General Planning and Programming Division of the Chief of Naval Operations.[3] The CNA models, which use input-output analysis, are applied to the study of cost allocations in the Navy. As developed at CNA, these models have proved to be of value for examining costs not only for mission and support categories but also for the program elements of the Five Year Defense Program (FYDP). Study of these models and discussion with their designers showed that there was a close enough relationship between the CNA and OCMM models so that it was possible to proceed with developments that offered possible further liaison between these two approaches. As a start in these developments, we may observe that the Markov matrices embedded in the OCMM model and the resource transfer rates of an input-output model, such as the one developed by CNA, share the so-called Minkowski-Leontief property in common. Each element of the input-output table is non-negative and each row sums to one in the same manner as a Markov matrix. Also, from a practical viewpoint, these input-output elements can be computed from relatively easily obtained historical data. The goal programming model which we shall present incorporates these strengths of the CNA input-output model. The result is a new "multi-level" model which integrates the resource allocation features of the CNA model with a career management model. Here, however, the two are considered simultaneously with resource allocation being accorded greater weight than career management in the objective of the resulting model. Goal programming features are also retained in order to deal with possible inconsistencies in resource availabilities and other requirements. This model is multi-level in the sense that two different levels of decison making are included in the same model.

In this model the input-output transfer rates provide an ability to examine simultaneously relationships between resource producers (e.g., the naval shore establishment) and final resource users (e.g., the fleet). The model also uses the generalized network structure with embedded Markov processes discussed in (7), to account for interactions between manpower "inventories" and manpower "requirements." The latter are joined together by a series of "coupling conditions" so that imbalances in one of the systems will be reflected in the other. The goal programming aspects of this model[4] also supply flexibility for examining the effects on final user demands of changes in resource inputs as well as manpower requirements.[5]

Data availability formed a very important consideration in the construction of this model. In fact, the model brings together two very large scale systems and the data necessarily had to come from existing or planned information systems. This pointed to the CNA models which use Navy Cost Information System (NCIS) data tapes reflecting the Navy's portion of the Five Year Defense Program (FYDP). Additionally, the model was designed to rely upon already developed computer programs to obtain the manpower transition

[1] Other possibilities such as man-job matching and job redesign are not covered. See, e.g. (10).
[2] See (7).
[3] See Augusta, *et. al.*, in (1)–(3).
[4] See (9) and Chapter X of (5) for a discussion of goal programming.
[5] It turns out that the classic input-output model can essentially be made a special case of this goal programming when there is perfect balance between the inputs and the outputs.

rates from OCMM's Personnel Automated Data System (PADS). For computational reasons the model also was designed to use linear programming for solution purposes. Note, for instance, that even though this goal programming model is stochastic and non-linear, it is possible to convert it to a linear equivalent for optimization purposes. This feature is preserved even in the extensions essayed to accommodate risk-related considerations in the chance-constrained programming formulation given at the end of this paper. Access is therefore immediate to all of the readily available solution routines and sensitivity checks associated with linear programming.

Static Goal Programming Model

A static version of the goal programming model with input-output elements is now given first so that the differences between it and the dynamic version will be apparent. A verbal description of the model structure is given in Figure 1. In this model, the objective is to minimize the weighted deviations from the final user requirements so that in goal programming form these are to be met "as closely as possible." In this case the deviations are accorded relative weights which reflect priorities associated with being over or under each of the final user requirements. These relative weights, which replace the dollar cost normally associated with conventional linear programming models, can be considered a "priority cost" of each of the final user requirements where the highest relative cost is associated with the most critical requirement.

OBJECTIVE: MINIMIZE COST (MEASURED BY RELATIVE PRIORITIES) OF BEING OVER/UNDER FINAL USER SUPPORT REQUIREMENTS

SUBJECT TO CONSTRAINTS OF:

TOTAL AMOUNT EACH FINAL USER SUPPORTED − POSSIBLE AMOUNT OVER + POSSIBLE AMOUNT UNDER = EACH FINAL USER SUPPORT REQUIREMENT

− TOTAL AMOUNT EACH FINAL USER SUPPORTED + SUM OF (PROPORTION OF OUTPUT EACH PRODUCER PROVIDES TO EACH FINAL USER X TOTAL OUTPUT OF EACH PRODUCER) = 0

TOTAL OUTPUT OF EACH PRODUCER ⩽ BUDGET OF EACH PRODUCER

SUM OF (CIVILIAN MANPOWER REQUIRED FOR EACH UNIT OF SUPPORT X TOTAL OUTPUT OF EACH PRODUCER) ⩽ TOTAL CIVILIAN MANPOWER AVAILABLE

Figure 1. STATIC GOAL PROGRAMMING–MODEL STRUCTURE

The remainder of the model structure is concerned with the various goal requirements and resource constraints which must be considered while trying to minimize goal discrepancies. The first group of conditions is concerned with setting the goals. This is accomplished by setting up an equation for each final user for each time period which states that the total amount of support furnished each user less the amount over plus the amount under will be equal to the total requirement, or goal. In any solution one will obtain for each equation the level of output associated with the goal and either the overage or underage (or zero deviation if the goal is met right on). This stems from the fact that one cannot be both over and under a goal at the same time. It should be noted at this point that all outputs are expressed in their dollar equivalents so that one can be consistent in adding the outputs from each of the producers.

The next set of conditions involves ensuring that the distribution of output from each of the producers is in the right proportion to the requirements of the final users. One can simultaneously calculate the support-on-support requirements or, alternatively, as is done here, a second stage allocation process may be used by means of the definitional relations. The latter is accomplished by using data from an input-output table which indicate for each producer the proportion of output required to support the final users. This ensures that the final users will be supported at the level required in the model solution and at the same time obtain the total amount of each output which should be produced. These relationships are built into the model by specifying an equation for each final user which states that the total amount of each output from each producer times the proportion used minus the total requirements from all producers will equal zero. This forces all the individual pieces of output from each type of support to be in the right proportion to the total.

The next section of the model bounds the amount of each kind of output to be produced. This is accomplished by assigning an equation to each producer for each time period which prescribes a limit to the amount that can be produced. This ensures that no producer will exceed its budget in trying to meet the overall goal. At the same time if an excess budget has been allocated to the producer it will show up in a non-zero value for the corresponding slack variable.

The final section of the model consists of equations which relate producer manpower requirements to the total available manpower. In this example only the static manpower relationships are shown.[1] In this static example there is an equation for each time period which states that the manpower per unit of output for each producer times the amount produced must be less than or equal to the total manpower available.

For this numerical example four producers and two final users were included. Figure 2 contains the base data for this example. The first four rows expressed in millions of dollars indicate the amount of output services each of the using sectors consumes. (For example, Producer 1 provides 95 million dollars of services to itself, 120 million dollars to Producer 2, etc.) The last row contains the amount of manpower required by each of the producers (e.g., Producer 1 requires 60,000 men, and so on.) Neither Final User 1 nor Final User 2 has any manpower associated with it since this model is oriented towards obtaining support establishment requirements to meet final user demand.

The next step is to convert these base data into utilization rates for use on input-output formulation. As far as the different types of output are concerned, this is done for each user including producers (to later obtain support-on-support requirements) by dividing the amount consumed by each user by the total amount produced. For example, Producer 1 consumes 95/920th or 10.33% of its own output, Producer 2 consumed 120/920ths or 13.04% of Producer 1's output, etc. The full array of these output usage rates are contained in Figure 3. The manpower usage rates are obtained by dividing the total manpower of a producer by the total amount produced. Thus, in this example, dividing the 60,000 men of Producer 1 by 920 million dollars yields 65.22 men per million dollars.

Three alternatives will be developed to show the reaction of the model to changing data conditions. In all these examples the final user requirements will be assumed to be constant. However, it will be assumed that there will be 5% inflation in wage and related manpower costs in Period 1 and 4% in Period 2. The inflation rate is included by decreasing the manpower usage rates by the appropriate factor, In this way one is receiving less manpower per million dollars of output. As a note of caution one should also be sure to increase the amount of final user demand in dollars to compensate for the fact that one is obtaining "less bang for the buck."

The non-constant input data for the three alternatives are given in Figure 4. It should be noted that the goals and budgetary levels of the producers are not in strict proportion to the historical distributions of outputs. The goals in the second period are both lower and in a different proportion to the first period goals. Considering this, and the effects of inflation, one will expect not only that the producer budget levels will decline but also that the required manpower will decrease at an even greater rate.

[1]The dynamic extension to include the relationships of manpower requirements to manpower inventories will be covered in the next section of this paper.

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4	FINAL USER 1	FINAL USER 2	TOTAL
PRODUCER 1	95	120	80	200	125	300	920 Million Dollars
PRODUCER 2	400	950	320	80	1,400	1,000	4,150
PRODUCER 3	100	150	75	250	1,050	400	2,025
PRODUCER 4	1,000	250	400	2,000	800	1,000	5,450
TOTAL	1,595	1,470	875	2,530	3,375	2,700	12,545
Civilian Manpower	60,000	12,000	23,000	200,000			295,000 Men

Figure 2. HISTORICAL USAGE DATA

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4	FINAL USER 1	FINAL USER 2
PRODUCER 1	.1033	.1304	.0870	.2174	.1359	.3260
PRODUCER 2	.0964	.2289	.0771	.0193	.3373	.2410
PRODUCER 3	.0494	.0741	.0370	.1235	.5185	.1975
PRODUCER 4	.1835	.0459	.0734	.3669	.1468	.1835
Civilian Manpower Per Million Dollars Output	65.22	2.89	11.36	36.70		

Figure 3. HISTORICAL USAGE DATA

Final User Requirements Assumed Constant for all Alternatives;
5% Inflation in Period 1, 4% Inflation in Period 2.

ALTERNATIVE 1: Heavy Budget Cuts
ALTERNATIVE 2: No Budget Increases
ALTERNATIVE 3: Substantial Budget Increases

MANPOWER PER MILLION DOLLARS OUTPUT

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4	REMARKS
PERIOD 1	61.88	2.75	10.79	34.87	5% Inflation
PERIOD 2	59.40	2.64	10.36	33.47	Additional 4% Inflation

FINAL USER SUPPORT REQUIREMENTS

	PERIOD 1	PERIOD 2
FINAL USER 1	3,375	3,200
FINAL USER 2	2,700	2,600

PRODUCER BUDGETARY AND MANPOWER LEVELS

	ALTERNATIVE 1		ALTERNATIVE 2		ALTERNATIVE 3	
	PERIOD 1	PERIOD 2	PERIOD 1	PERIOD 2	PERIOD 1	PERIOD 2
PRODUCER 1	875	830	920	920	1,200	1,200
PRODUCER 2	3,940	3,740	4,150	4,150	4,500	4,500
PRODUCER 3	1,925	1,830	2,025	2,025	2,500	2,500
PRODUCER 4	5,175	4,920	5,450	5,450	6,000	6,000
CIVILIAN MANPOWER	295	295	295	295	295	295

Figure 4. ALTERNATIVE INPUT DATA

The data are next arranged into a format which facilitates the development of linear programming features of the model. The linear programming input matrix for a two-period model which facilitates comparative coordination while maintaining a purely static situation is given in Figure 5. With this in mind the relative priorities have been assumed to be equal and set to one in all cases.

The solutions to the three alternatives are given in Figure 6. In the first alternative, of heavy budget cuts, the final user requirements were not met completely. Also, there was a sizable amount of unused manpower along with the fact that the budgets of the producers were used in their entirety. In the second alternative, of no budget increases, the final user requirements were met. However, since inflation was operating, the total manpower strength must be reduced by 13 thousand men in the first peirod. Also, one is not

	COLUMN NAME / ROW NAME	POS DESCREP				NEG DESCREP				USER OUTPUT				PRODUCER OUTPUT LEVELS								SIGN	RHS
		PDUS 11	PDUS 21	PDUS 31	PDUS 41	NDUS 12	NDUS 22	NDUS 32	NDUS 42	USER 11	USER 21	USER 12	USER 22	PROD 11	PROD 21	PROD 31	PROD 41	PROD 12	PROD [illegible]	PROD 32	PROD 42		
RELATIVE PRIORITIES	RELPRI	1	1	1	1	1	1	1	1														
FINAL USER GOALS	GUSE 11	-1				1				1												=	3375
	GUSE 21		-1				1				1											=	2700
	GUSE 12			-1				1				1										=	3200
	GUSE 22				-1				1				1									=	2600
USAGE RATES	RUSE 11									-1				.1359	.3373	.5185	.1468					=	0
	RUSE 21										-1			.3260	.2410	.1975	.1835					=	0
	RUSE 12											-1						.1358	.3373	.5185	.1468	=	0
	RUSE 22												-1					.3260	.2410	.1975	.1835	=	0
PRODUCTION LEVELS	LPRO 11													1									875
	LPRO 21														1								3940
	LPRO 31															1							1925
	LPRO 41																1						5175
	LPRO 12																	1					830
	LPRO 22																		1				3740
	LPRO 32																			1			1830
	LPRO 42																				1		4920
MANPOWER CONSTRAINTS	MAN 1													.0619	.0028	.0108	.0349						295
	MAN 2																	.0594	.0026	.0104	.0335		295

Figure 5. STATIC GOAL PROGRAMMING MODEL–LINEAR PROGRAMMING MATRIX

FINAL USERS

	Alternative 1			Alternative 2			Alternative 3		
	REQ.	ACT.	DIFF.	REQ.	ACT.	DIFF.	REQ.	ACT.	DIFF.
PERIOD 1									
F. U. 1	3,375	3,206	-169	3,375	3,375		3,375	3,375	
F. U. 2	2,700	2,565	-135	2,700	2,700		2,700	2,700	
PERIOD 2									
F. U. 1	3,200	3,045	-155	3,200	3,200		3,200	3,200	
F. U. 2	2,600	2,436	-164	2,600	2,600		2,600	2,600	

PRODUCER BUDGETARY AND MANPOWER RESOURCES

	Alternative 1			Alternative 2			Alternative 3		
	Available	Used	Unused	Available	Used	Unused	Available	Used	Unused
PERIOD 1									
PRODUCER 1	875	875		920	920		1,200	1,200	
PRODUCER 2	3,940	3,940		4,150	4,150		4,500	4,500	
PRODUCER 3	1,925	1,925		2,025	2,025		2,500	1,982	518
PRODUCER 4	5,175	5,175		5,450	5,450		6,000	4,538	1,462
CIV. MAN.	295	268	27	295	282	13	295	268	27
PERIOD 2									
PRODUCER 1	830	830		920	920		1,200	1,200	
PRODUCER 2	3,740	3,740		4,150	4,150		4,500	4,500	
PRODUCER 3	1,830	1,830		2,025	1,763	262	2,500	1,720	780
PRODUCER 4	4,920	4,920		5,450	5,185	265	6,000	4,273	1,727
CIV. MAN.	295	256	39	295	263	32	295	244	51

Figure 6. STATIC MODEL SOLUTION DATA

receiving as much physical output since the dollars are worth 5% less. In the second time period, the manpower strength must be reduced because of the fact that an additional 4% inflation is present and from the fact that less final user demand must be filled. Additionally, there are unused dollars by some of the producers since less demand must be satisfied. In the third alternative where there are substantial budget increases one again finds the final user demand satisfied. However, there are more unused dollars by the producers. It also appears that the model is favoring one producer over another when a choice is present.

The support-on-support requirements can now be calculated. They are obtained by simply multiplying the resultant producer budget levels by the corresponding row of rates in the input-output table. These data are shown in Figure 7. For example, in Alternative 1 the resultant budget level of 875 million dollars for Producer 1 is multiplied by the utilization rate of 0.1033 to obtain the 90 million dollars which Producer 1 uses to support itself.

The static model does not include all of the characteristics one might want when modelling the area of internal strategic planning. For one thing, the model does not relate one period to another or take into account manpower inventories already on board. Also, in situations where flexibility is possible in the

Alternative 1

		PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PRODUCER 1	PERIOD 1	90	114	76	190
PRODUCER 2	PERIOD 1	380	902	304	76
PRODUCER 3	PERIOD 1	95	143	71	238
PRODUCER 4	PERIOD 1	950	238	380	1,899
PRODUCER 1	PERIOD 2	86	108	72	180
PRODUCER 2	PERIOD 2	361	856	288	72
PRODUCER 3	PERIOD 2	90	136	68	226
PRODUCER 4	PERIOD 2	903	226	361	1,658

Alternative 2

		PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PRODUCER 1	PERIOD 1	95	120	80	200
PRODUCER 2	PERIOD 1	400	950	320	80
PRODUCER 3	PERIOD 1	100	150	75	250
PRODUCER 4	PERIOD 1	1,000	250	400	2,000
PRODUCER 1	PERIOD 2	95	120	80	200
PRODUCER 2	PERIOD 2	400	950	320	80
PRODUCER 3	PERIOD 2	87	131	165	218
PRODUCER 4	PERIOD 2	951	238	380	1,900

Alternative 3

		PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PRODUCER 1	PERIOD 1	124	156	104	261
PRODUCER 2	PERIOD 1	434	1,030	347	87
PRODUCER 3	PERIOD 1	98	147	73	245
PRODUCER 4	PERIOD 1	833	208	333	1,665
PRODUCER 1	PERIOD 2	124	156	104	261
PRODUCER 2	PERIOD 2	434	1,030	347	87
PRODUCER 3	PERIOD 2	85	127	64	212
PRODUCER 4	PERIOD 2	784	196	314	1,568

Figure 7. SUPPORT-ON-SUPPORT REQUIREMENTS–STATIC MODEL

allocation of resources among producers, the favoring of one producer over another appears to require more control than available in the present configuration. Some of these problems might be overcome by such measures as fixing lower bounds on the budgetary levels. Additionally, the required control might be obtained by setting the relative priorities appropriately. However, at this point it might be worthwhile to turn to the dynamic model to see how it reacts to the same input data.

Dynamic Multi-Level Model

A verbal description of the dynamic model is given in Figure 8. As can be seen, the top half for the model structure is the same as the static model. The bottom half consists of a "generalized network"[1] structure to accommodate the relationships between manpower requirements and manpower inventories over time. These two model systems are coupled together by means of a set of equations which relate the amount of manpower required by each producer for a given level of final user demand.

OBJECTIVE: MINIMIZE COST (MEASURED BY RELATIVE PRIORITIES) OF BEING OVER/UNDER FINAL USER SUPPORT REQUIREMENTS *AND* OF RECRUITING/REDUCTIONS OF CIVILIAN MANPOWER

SUBJECT TO CONSTRAINTS OF:

TOTAL AMOUNT EACH FINAL USER SUPPORTED − POSSIBLE AMOUNT OVER + POSSIBLE AMOUNT UNDER = SUPPORT REQUIREMENT

− TOTAL AMOUNT EACH FINAL USER SUPPORTED + SUM OF (PROPORTION OF OUTPUT EACH PRODUCER PROVIDES TO EACH FINAL USER X TOTAL OUTPUT OF EACH PRODUCER) = 0

TOTAL OUTPUT OF EACH PRODUCER ≤ BUDGET OF EACH PRODUCER

SUM OF (CIVILIAN MANPOWER REQUIRED FOR EACH UNIT OF SUPPORT X TOTAL OUTPUT OF EACH PRODUCER) − REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER = 0

REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER ≤ TOTAL CIVILIAN MANPOWER AVAILABLE

CIVILIAN MANPOWER ON-BOARD AT START = INITIAL POPULATION

− REQUIRED CIVILIAN MANPOWER PROVIDED BY ON-BOARD MANPOWER FROM PREVIOUS PERIODS + CIVILIAN MANPOWER ON-BOARD AT PRESENT PERIOD − REQUIRED CIVILIAN MANPOWER PROVIDED BY NEW HIRES + EXCESS CIVILIAN MANPOWER = 0

Figure 8. DYNAMIC MULTI-LEVEL MODEL STRUCTURE

[1] See (7).

The generalized network portion of the model uses manpower transition rates in a Markov matrix structure. First, the number on board at the start is set equal to a constant. This ensures that the base period population will be completely accounted for in the model solution. In the first period of the forecast, the base period population is multiplied by the transition matrix to obtain the number remaining as well as the internal transfers. These, plus any "New Hires," or less any "Excess Manpower," must exactly equal the on-board manpower at the end of the first period. This ending population is again subjected to the same process in the next time period. This "walking of oneself forward" in time continues until an accounting has been made for all of the time periods to be included in the forecast.

A second set of relative priorities must be included for the new hire and excess manpower columns of the model. These are needed to ensure that the model will first choose on-board manpower before hiring or releasing manpower. The values of these relative priorities must, as a group, be set higher than those on the goal discrepancies of the final user demands will in fact drive the model.

The data in Figure 9 were developed so that the static example might be reconfigured into a dynamic structure and thereby aid in drawing the comparisons that we shall make. What is given here are data on two categories of manpower-White Collar and -Blue Collar. The manpower rates for each of the producers per million dollars of output are given and include the inflation factors used in the static example. Additionally, a transition matrix is given which describes the internal movement and attrition from the work force. The linear programming input matrix is given in Figure 10. Here, the relative priorities have been set to one for the final user goal discrepancies and to two for the new hire and excess manpower categories.

The solution data are given in Figures 11. 12, and 13. In all of the alternatives the manpower used is significantly different from the static examples. This is due to the fact that the dynamic model is sensitive to manpower changes since a penalty must be paid to hire or fire personnel. Additionally, the dynamic interactions of the second period manpower requirements to the manpower inventories from the first period are apparent. In the first alternative the amount of final user support from the producers remained the same with all of the producers budgets consumed. This is not true for the second and third alternatives. They clearly demonstrate that the transition rates operate at least qualitatively in a manner which corresponds to what the model design is expected to produce.

Support-on-support requirements as shown in Figure 13 can again be generated. As before, one simply multiplies the resultant budget levels by the appropriate rows of the input-output matrix. For example, in Alternative 2, the resultant budget levels of 920 million dollars for Producer 1 is multiplied by 0.1304 to obtain the 120 million dollars of support which Producer 1 must provide to Producer 2.

Model Uses and Extensions

The numerical examples were meant to show the basic model structure and how alternative solutions may in fact be obtained. The examples also provide some idea as to the model uses. The allocations obtained from the static model suggest that multi-period analyses should be obtained from dynamic models and that the use of input-output alone is too restrictive for managerial planning when unbalanced resource inputs must be considered. The importance of the input-output structure embedded in the model is the consistency which can be obtained between the various resource inputs and production outputs. The input-output matrix also provides the transfer table which allows the coupling of strategic requirements planning with the upper bounds or budgets of the producers.

The relaxation of strict balancing requirement such as is not associated with input-output analysis does not in any way hamper the usual "feasibility and consistency" checks of input-output studies. Indeed, these are facilitated since feasibility conditions are explicitly incorporated in terms of stipulated constraints and inconsistencies are explicitly delineated by the resulting goal programming manipulations. This provides added flexibility in dealing, for example, with the possible inconsistencies between output goals and resource inputs.

BASE PERIOD ON-BOARD

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
White	40,000	10,000	18,000	60,000
Blue	20,000	2,000	5,000	140,000
Total	60,000	12,000	23,000	200,000

MANPOWER PER MILLION DOLLARS OUTPUT—BASE PERIOD

	PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
White	43.48	2.41	8.89	11.01
Blue	21.74	0.48	2.47	25.67
5% Inflation One Period Later				
White	41.48	2.29	8.44	10.46
Blue	20.40	0.46	2.35	24.41
5% Inflation One Period Later; 4% Inflation Two Periods Later				
White	39.82	2.20	8.10	10.04
Blue	19.58	0.44	2.26	23.43

TRANSITION RATES (Read Down)

	White	Blue
White	.90	.05
Blue		.80

Figure 9. DYNAMIC MODEL MANPOWER DATA

Of course, other possible extensions and reformulations are also possible. Observe, for instance, how dynamic form alters the need for controlling New Hires in the static case. This is a result of the fact the on-board manpower is favored over recruiting or reductions. This follows from the fact that the dynamic model requires a penalty to be paid in the form of an increase in the objective function when recruiting or reductions are included in the solution. The dynamic model also provides additional control in the form of balancing of short-run (or period by period) and long-run requirements. It can be expected that the need for any additional features in the model will become understood better when larger numerical examples are tested using operational data.

We shall shortly exhibit how further extensions can be effected when risk-related controls are also to be incorporated in these models. First, however, we may pause to summarize at least some of the potential management uses of the preceding model. These include:

1. Ways for evaluating the impact on manpower and other resource requirements of additions to or deletions from final user support requirements.

	POSITIVE GOAL DISCREP.	NEGATIVE GOAL DISCREP.	FINAL USER SUPPORT	PRODUCER OUTPUT	ON-BOARD MANPOWER	NEW HIRE MAN-POWER	EXCESS MAN-POWER	SIGN	RHS
RELATIVE PRIORITIES	α	α				β	β		
FINAL USER GOALS	-1 -1	1 1	1 1					= =	FINAL USER SUPPORT REQUIREMENTS
SUPPORT USAGE RATES			-1 -1	I-O RATES I-O RATES				= =	0 0
PRODUCTION LIMITS				1 1				$\leq$ $\leq$	PRODUCER BUDGETARY LIMITS
PRODUCER OUTPUT/ MANPOWER RATES				$(m/\$)_1$ $(m/\$)_2$	-1 -1			= =	0 0
TOTAL MANPOWER CONSTRAINTS					1111 1111			$\leq$ $\leq$	TOTAL MANPOWER AVAILABLE
MANPOWER ATTRITION					1 -M 1 -M 1	-1 -1	1 1	= = =	INIT. POP. 0 0

Figure 10. DYNAMIC MULTI-LEVEL MODEL–LINEAR PROGRAMMING MATRIX

FINAL USERS

	Alternative 1			Alternative 2			Alternative 3		
	REQ.	ACT.	DIFF.	REQ.	ACT.	DIFF.	REQ.	ACT.	DIFF.
PERIOD 1									
F. U. 1	3,375	3,206	-169	3,375	3,375		3,375	3,375	
F. U. 2	2,700	2,565	-135	2,700	2,700		2,700	2,700	
PERIOD 2									
F. U. 1	3,200	3,045	-155	3,200	3,200		3,200	3,200	
F. U. 2	2,600	2,436	-164	2,600	2,600		2,600	2,600	

SUPPORT ELEMENTS—PRODUCER BUDGETARY AND MANPOWER RESOURCES

	Alternative 1			Alternative 2			Alternative 3		
	Available	Used	Unused	Available	Used	Unused	Available	Used	Unused
PERIOD 1									
PRODUCER 1	875	875		920	920		1,200	892	308
PRODUCER 2	3,940	3,940		4,150	4,150		4,500	4,500	
PRODUCER 3	1,925	1,925		2,025	2,025		2,500	1,876	624
PRODUCER 4	5,175	5,175		5,450	5,450		6,000	5,200	800
MANPOWER	295	264	31	295	278	17	295	267	28
PERIOD 1									
PRODUCER 1	830	830		920	886	34	1,200	859	341
PRODUCER 2	3,740	3,740		4,150	4,150		4,500	4,500	
PRODUCER 3	1,830	1,830		2,025	1,751	274	2,500	1,601	899
PRODUCER 4	4,920	4,920		5,450	5,258	192	6,000	5,007	993
MANPOWER	295	241	54	295	256	39	292	260	32

Figure 11. DYNAMIC MODEL SOLUTION DATA

2. The provision of an explicitly delineated structure for making resource allocation decisions and observing potential discrepancies.

3. Systematically supplied ways for evaluating inconsistencies between manpower and budgetary allocation decisions.

4. Systematic ways for evaluating effect of inflation or other such changes in operating force support requirements and manpower requirements.

MANPOWER DATA

		Alternative 1			Alternative 2			Alternative 3		
		On-Board	New Hire	Excess	On-Board	New Hire	Excess	On-Board	New Hire	Excess
PERIOD 1										
PRODUCER 1	White	36.3		0.7	38.2	1.2		37.1		
	Blue	17.9	1.9		18.8	2.8		18.2	2.2	
PRODUCER 2	White	[illegible]		0.1	9.5	0.4		10.3	1.2	
	Blue	1.8	0.2		1.9	0.3		2.1	0.5	
PRODUCER 3	White	16.2		0.2	17.1	0.6		15.8		0.6
	Blue	2.2		1.8	2.3		1.7	2.1		1.9
PRODUCER 4	White	54.1		6.9	57.0		4.0	54.4		6.6
	Blue	126.3	14.3		133.0	21.0		126.9	14.9	
PERIOD 2										
PRODUCER 1	White	33.1		0.5	35.2			34.2		
	Blue	16.3	2.0		17.3	2.3		16.8	2.3	
PRODUCER 2	White	8.2			19.1	0.5		9.9	0.5	
	Blue	1.6	0.2		1.8	0.3		2.0	0.3	
PRODUCER 3	White	14.8	0.1		14.2			13.0		1.4
	Blue	1.9	0.2		1.9	0.1		1.7		
PRODUCER 4	White	49.4		5.6	52.8		5.2	50.3		5.0
	Blue	115.3	14.2		123.2	16.8		117.3	15.8	

Figure 12. DYNAMIC MODEL SOLUTION DATA

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Alternative 1—Same as Static Model

Alternative 2

		PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PRODUCER 1	PERIOD 1	95	120	80	200
PRODUCER 2	PERIOD 1	400	950	320	80
PRODUCER 3	PERIOD 1	100	150	75	250
PRODUCER 4	PERIOD 1	1,000	250	400	2,000
PRODUCER 1	PERIOD 2	92	116	77	193
PRODUCER 2	PERIOD 2	400	950	320	80
PRODUCER 3	PERIOD 2	86	130	65	216
PRODUCER 4	PERIOD 2	965	241	386	1,929

Alternative 3

		PRODUCER 1	PRODUCER 2	PRODUCER 3	PRODUCER 4
PRODUCER 1	PERIOD 1	92	116	78	194
PRODUCER 2	PERIOD 1	434	1,030	347	87
PRODUCER 3	PERIOD 1	93	139	69	232
PRODUCER 4	PERIOD 1	954	239	382	1,908
PRODUCER 1	PERIOD 2	89	112	75	187
PRODUCER 2	PERIOD 2	434	1,030	347	87
PRODUCER 3	PERIOD 2	79	119	59	198
PRODUCER 4	PERIOD 2	919	230	368	1,837

Figure 13. SUPPORT-ON-SUPPORT REQUIREMENTS—DYNAMIC MODEL

5. Determination and depiction of the effects of attrition and internal manpower transfers on both short and long run decisions.

6. The integration of career management, training, recruitment and advancement planning with budgetary and strategic decisions.

Present planning is to substitute the multi-level model structure for the projection portions of the Five Year Navy Civilian Manpower Plan (FYNCIMP) system. This will be done in such a way that one can obtain projections of: (a) the career management dynamics unconstrained by financial limitations or (b) the manpower dynamics constrained by the resource allocation portion of the model. Small manually developed examples using actual CNA and OCMM data have already been tested in order to design this revised FYNCIMP system. Once this conversion has been accomplished current information requirements as well as the data needed for testing of large scale examples of the multi-level model will be able to be supported from the same information system.

Addendum

It is not customary to interpret input-output matrices in a truly stochastic (i.e., probabilistic) manner. Nevertheless, these matrices admit of such interpretations and many of the goal programming applications for manpower planning suggest such orientations. Since the input-output concepts are here also joined to those of goal programming, it therefore seems prudent to extend the tradition of such developments as were detailed,

for instance, in (7), so that these, too, admit of use on such probabilistic contacts. This we shall now do in terms of a sketch which is tailored to remain within the space allowances of this addendum. Having previously extended the Markoff matrix notations of the OCMM models to chance-constrained interpretations (see (7)), we shall proceed in a complementary manner to consider chance-constrained applications to deal with risk variations in the right hand sides.

The elements we single out for specific illustrative treatment involve the final user goals and the producer budgetary limitations. Therefore, refer to Figure 14 which generalizes Figure 10 by introducing the indicated stochastic model elements. Here we restrict these chance-constrained extensions, as in (7), although other extensions–separately or jointly–may also be effected as in (4).[1]

A use of chance-constraints naturally introduces certain new elements. These are elaborated and defined in Figure 14. For an explanation and illustration, we may begin by considering the "k th" Producer Budgetary Limitation;

$$P\left[(PO)_k \leqslant (PBL)_k\right] \geqslant \gamma_{B_k}$$

Objective: Minimize the weighted deviations from Final User Requirements plus the costs of New Hires and Excess Manpower

A. Subject to constraints:

P (Final User Support $\leqslant$ Final User Requirements + δ^+) $\geqslant \gamma_G$

P (Final User Support $\geqslant$ Final User Requirements − δ^-) $\geqslant \gamma_G$

P (Producer Output $\leqslant$ Producer Budgetary Limitation) $\geqslant \gamma_B$

P (On-Board manpower + New Hires $\leqslant$ Total Manpower Available) $\geqslant \gamma_M$

where

$P(X \leqslant t) \equiv F_x(t)$ which is the probability that the variable X will not exceed t

and γ, suitably subscripted, is a vector of minimum levels of probability at which the chance constraints must hold

Hence, the random variables are:

$(FUR)_i$ = the "i th" final user requirement

$(PBL)_k$ = the "k th" producer budgetary limitation

$(TMA)_j$ = the "j th" total manpower availability

The corresponding marginal distribution functions are F_{G_i}, F_{B_k}, and F_{M_j}.

The δ_i^+, δ_i^-, are the planned goal deviations which are also weighted in the function.

B. Subject also to the other constraints of Figure 10.

Figure 14. CHANCE-CONSTRAINED EXTENSION OF MULTI-LEVEL MODEL

[1] Such extensions, we may note, will be crucial even for applications in the strictly military component when, for instance, considerations like an all-volunteer force (or even the volunteer component of a conscripted force) are important elements for planning purposes.

This means that the "k th" Producer Output must be chosen so that it does not exceed the "k th" Producer Budgetary Limitation with a probability of at least γ_{B_k}. Note then that $(PBL)_k$ is a random variable whose sample value is not known when the planning decision for $(PO)_k$ must be made. It is required, however, that $(PO)_k$, when selected, must not exceed $(PBL)_k$ by the probability γ_{B_k} which is also stipulated prior to knowledge of the sample value of $(PBL)_k$. Only the probability distribution for $(PBL)_k$ is known when the planning decisions are to be made.

The U. S. Navy's planning context calls for 5-year projections of planning decisions as part of a general PPBS (Program Planning Budgeting System) for directing and coordinating all elements of the total military plan.[1] In this case, each year's projections are made without considering the effect of the previous year's projections. The natural analytic correspond in the theory of chance-constrained programming is a "zero-order decision rule." Under this rule the values of the decision variables are all chosen in advance of knowledge of the sample values of the random variables.

With the zero-order decision rules, the chance constraints may be inverted to obtain a new linear programming problem of virtually the same structure as the preceding deterministic case. For example, the Producer Budgetary Limitation Chance Constraint can be inverted to yield the following inequality

$$(PO)_k \leq F_{B_k}^{-1}\left(1 - \gamma_{B_k}\right)$$

where $F_{B_k}^{-1}$ is the inverse function for F_{B_k} (as defined in Figure 14). From an inspection of this last expression, however, we can see that it differs from the corresponding constraint in Figure 10 only in the right-hand side. (I.e., the $F_{B_k}^{-1}\left(1 - \gamma_{B_k}\right)$ replaces the $(PBL)_k$ of that Figure.) Similarly, the "j th" chance constraint for the On-Board and New Hire Manpower inverts to:

$$(OBM)_j + (NHM)_j \leq F_{M_j}^{-1} \; 1 - \gamma_{M_j} .$$

This, again, differs from its Figure 10 correspond by replacement of the "j th" Total Manpower Availability by

$$F_{M_j}^{-1}\left(1 - \gamma_{M_j}\right).$$

Continuing,

$$s_i^- - \delta_i^+ + (FUS)_i = F_{G_i}^{-1}\left(1 - \gamma_{G_i}\right)$$

$$-s_i^+ + \delta_i^- + (FUS)_i = F_{G_i}^{-1}\left(\gamma_{G_i}\right)$$

replaces the constraints for Final User Goals in Figure 10. Again the "i th" pair of Final User Requirements is replaced by the above right-hand sides. Note, however, that new slack variables s_i^+, s_i^-, have been introduced in order to accommodate the modelling of the nonlinear goal litions by the linear chance constraints in a very natural extension of the ideas of goal programming.[2]

We now systematize and complete this development by extending Figure 10 to the one we now portray as Figure 15. Here the symbols $F_{G_i}^{-1}\left(1 - \gamma_{G_i}\right)$ and $F_{G_i}^{-1}\left(\gamma_{G_i}\right)$ refer to vectors with components as indicated in Figure 14, and similar remarks apply to $F_{B_k}^{-1}\left(1 - \gamma_B\right)$ and $F_{M_j}^{-1}\left(\; - \gamma_{M_j}\right)$.

[1] I.e., the so-called Five Year Defense Plan (FYDP) of the U. S. Department of Defense.
[2] See Chapter X in (5).

	POSITIVE SLACK	NEGATIVE SLACK	POSITIVE GOAL DISCREP.	NEGATIVE GOAL DISCREP.	FINAL USER SUPPORT	PRODUCER OUTPUT	ON-BOARD MANPOWER	NEW HIRE MAN-POWER	EXCESS MAN-POWER	SIGN	RHS
RELATIVE PRIORITIES			α	α				β	β		
FINAL USER GOALS		I	-I	I	I					=	$F^{-1}_{G_1}(1-\gamma_{G_i})$
	-I		-I	I	I					=	$F^{-1}_{G_1}(\gamma_{G_i})$
		I	-I	I	I					=	$F^{-1}_{G_2}(1-\gamma_{G_2})$
	-I		-I	I	I					=	$F^{-1}_{G_2}(\gamma_{G_2})$
SUPPORT USAGE RATES					-I	I-O RATES				=	0
					-I	I-O RATES				=	0
PRODUCTION LIMITS						I				$\leq$	$F^{-1}_{B_1}(1-\gamma_{B_1})$
						I				$\leq$	$F^{-1}_{B_2}(1-\gamma_{B_2})$
MANPOWER PRODUCER OUTPUT						$(m/\$)_1$	-I			=	0
						$(m/\$)_2$	-I			=	0
TOTAL MANPOWER CONSTRAINTS							1111			$\leq$	$F^{-1}_{M_1}(1-\gamma_{M_1})$
							1111			$\leq$	$F^{-1}_{M_2}(1-\gamma_{M_2})$
MANPOWER ATTRITION							I			=	INITIAL POP.
							-M I	-I	I	=	0
							-M I	-I	I	=	0

Figure 15. CHANCE-CONSTRAINED EXTENSION OF MULTI-LEVEL MODEL—LINEAR PROGRAMMING MATRIX

Now, we may remark that no supposition of independence is required to obtain these reductions. Also, no supposition other than strict monotonicity of the marginal distribution functions have been made—and even this latter may be relaxed. A further ability of this model may also be noted—*viz.*, utilizing multimodal distributions to encompass unanticipated extreme values of the random variables, as corresponding, e.g., to emergency requirements which need to be considered in the planning even when their occurrence is unlikely. This follows from the fact that the above reductions do not depend on assumptions of uni-modality.

These and other topics, including extensions to higher order decision rules, etc., which form part of the continuing research program at OCMM, will not be reported here.

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Comments and Questions at NATO Conference on Manpower Planning Models
Cambridge, United Kingdom
5-10 September 1971

General Comment:

Our very recent completion of the formal model discussed above, made it impossible for us to assemble the data and obtain a solution to a large-scale problem prior to our leaving for this conference. Dr. Joel Stutz of the University of Texas, however, persevered and with extraordinary assistance from Dr. Gomez-Rivas of the Houston, Texas, Data Service Division of the Control Data Corporation was able to take a 5-year forecast from actual Navy data and drive it through to a solution on their CDC 6600 computer. These results, which were just received by cable, can be read into the record as follows:

L. P. Matrix Size: 3165 rows X 5265 columns with a density of 0.13%

Code Employed: Ophelie II

Phase I: 11 minutes; 4,035 pivots
Phase II: 5 minutes; 3,079 pivots
Peripheral Processor (Input-Output) Time: 1 hour; 250 pages

The 16 minute CPU time can surely be reduced by a factor of 3 or more since Phase I can be eliminated by providing a feasible start. A good start should also reduce the Phase II time and this should not be too difficult to arrange, especially as results of prior experience including parametric studies begin to be available.

COMMENTS BY S. VAJDA, United Kingdom

Who decides what levels of satisfaction to use for the chance-constraints, as reflected in the last column of the last table in the paper?

RESPONSE TO S. VAJDA

We assume that the reference for this question is to the "confidence (or risk) coefficients" although the "quality levels," too, need to be considered. We can perhaps best clarify this remark by writing a typical chance-constraint as follows:

$$P_r\left[\sum_{j=1}^{M} a_{ij}\, x_j \geq \lambda_i\, b_i + \delta_i\right] \geq \alpha_i.$$

Here α_i is a risk or confidence coefficient stipulated at a given level while $\lambda_i\, b_i + \delta_i$ is a quality level assigned to a random b_i via the parameters λ_i and δ_i. For instance, if we interpret b_i in terms of the demand for a product like heating oil,[1] known only as a random variable, then the above expression might verbalize as follows: assign values to the variables so that, say, λ_i = 80% of the quantity demanded, b_i, plus a stipulated safety amount, δ_i, will be satisfied at least 90% of the time–i.e., with probability at least $\alpha_i = 0.9$.

Even at the inception of chance constrained programming (at ESSO), it was necessary, or prudent, to provide for parametric analysis and evaluation of both the quality levels and the risk coefficients via variations in the α_i, λ_i, and δ_i, values. This is done, e.g., via the use of "deterministic equivalents," such as those displayed in the examples in the present paper and, naturally, provision is being made for those subsequent analyses and evaluations in these cases, too. As was true at Esso (and other applications) the point is to get a start with some values which appear reasonable and then to study the consequences of varying these values parametrically.

This latter step is best delayed, we think, until we can erect a suitable organization for such evaluations. At Esso this took the form of a "Risk Committee" which could look to responsibility for varying these levels in a product heavily charged with a public interest along with cost and other considerations. Something like this may need to be done here, too, but for the moment we need to make clear that we erected this model as a way of making contact between the OCMM series of models and other work in manpower planning at the U. S. Navy. In particular we sought to make contact in this manner with the resources management work via input-output analyses which Joseph Augusta and his associates are conducting at the Center for Naval Analyses.

The latter being the most pressing issue we have delayed pressing along the lines suggested by Dr. Vajda's questions although, as already indicated, provision has been made for studying and revising these risk and quality levels as conditions (including organization conditions) and the probl- may warrant.

[1]The problem of scheduling heating oil provided the context in which chance-constrained programming was invented at the Standard Oil Company of New Jersey research offices in Linden, New Jersey.

COMMENTS BY MR. W. L. PRICE, Canada

I am mainly interested in your computational experience, and in the stability of the solutions with respect to such things as the weights in the objective function, and variations of the RHS parameters. Another point that is of interest to me is that the monetary constraints are included directly in the model, rather than converting them to manpower levels previously. This increases the size of the solution space and could lead to unacceptable solutions if the weights are not suitably chosen.

Could you discuss further your method of generating the objective function weights?

RESPONSE TO W. L. PRICE

Part of the answer of this question was supplied in our immediately preceding response to Professor Vajda. The weights in the objective function presently being used should be regarded only as a start, as is true here also of the right-hand side parameters. Systematic procedures can be worked out for choosing all of these elements, as we noted above—and subsequent published literature by persons like B. Naslund, A. Geoffrion and others have now been directed to this specific problem in chance-constrained and goal programming context. More can be done still, of course, but of precise answers to these questions of weights, etc., are best delayed, we think, rather than attempting to attend to them in all detail at the start of an analysis. Remember, in any event, these models have mainly been used to date in special studies rather than in day-to-day decision making. Hence the weights must be chosen anew relative to the context of each such study and its particular objectives.

Current budgetary restrictions have made it highly desirable to deal with financial as well as manpower restrictions. Unacceptable solutions might arise from the financial side, too, if we focused only on manpower restrictions although, naturally, we might want to study the consequences of omitting or altering either the financial or the manpower limitations. This kind of consideration would seem to be more important than questions of expanding or contracting the mathematical spaces involved. Notice, for instance, that reducing the number of variables in a linear programming problem has sometimes been regarded as desirable. This can be erroneous since, for example, certain types of problems can be solved much more efficiently by increasing the number of variables. A case in point is the warehousing type of model where we have shown that the solution process and time can be drastically simplified and reduced by quadrupling the number of variables. The latter development required special mathematical innovations suggested by the structure of the solution matrices. Generally speaking, mathematical structure is more important than counts of variables or constraints.

COMMENTS BY J. A. CANNON, United Kingdom

Have you been able to investigate the relationships (if any) between transitions and characteristics (e.g., age, skill, length of service, size of category) of the population from which the transitions are made?

In particular are there any limiting conditions amongst these characteristics, i.e., how sensitive are the transitions to such factors?

RESPONSE TO J. A. CANNON

We have made some preliminary investigations of age and length-of-service effects on the transition rates. These have all resulted from only empirical studies, however, and much more remains to be done in theory development, etc., including recourse to behavioral sciences research at fairly deep and intense levels. More particularly, we would like to see a behavioral sciences research effort joined with these models as a way of systematizing such research and interacting with it in a decision-oriented context.

Some of the simpler phenomena have already been exploited, such as allowances for transition rate adjustments when people at the terminal periods are involved. Other more complex and subtle ones remain to be dealt with such as, e.g., the effect on transition rates when large-scale reductions-in-force are undertaken or when crash recruitment occurs on a large scale for new programs, etc. Such "context" effects clearly require something more than analyses of stable patterns from past history. On the other hand, the term "context" suggests something more is needed than mere elaboration of individual or group psychological findings from past research. Thus a team approach in a decision (i.e., problem) oriented context would seem to be an inviting way to proceed.

COMMENTS BY C. J. PURKISS, United Kingdom

The formulation of the programs assumes that transitions between jobs or groups of jobs are fixed a priori. I appreciate that these can be varied between time periods or for different runs of the models. To what extent is it possible within this particular formulation to include transfer between jobs, as a set of decision variables, so that the manager can get guidance on how best to redeploy his labor force during periods of changing demand for manpower?

RESPONSE TO C. J. PURKISS

This question is in advance of the present state of practice. A fully satisfactory answer would necessitate research of a behavioral science variety into reasons lying behind observed transition probabilities. On the other hand, this should not be interpreted to imply that only uncontrolled behavioral sciences research will itself be adequate. The systems context and the possibility of using *two*-way managerial decision making (including motivation, etc.), needs to be considered—e.g., in ways that have not been characteristic of past behavioral-sciences research in areas like industrial psychology.

We have discussed these and other matters in our paper "Mathematical Models for Manpower and Personnel Planning" which is now available in A. I. Siegel, ed., *Symposium on Computer Simulation as Related to Manpower and Personnel Planning* (Washington, D. C.: Naval Personnel Research and Development Laboratory, Washington Navy Yard 20390, July, 1971). The following quotation from that paper may also be apt for present purposes:

> "... consider the elements (i.e., the transition probabilities) of any such matrix which is being used in the indicated manner. These matrix elements also have important managerial consequences and hence invite attention for further research and potential applications in ways that do not appear to have been explored heretofore. What types of behavioral science research should be undertaken, for instance, to obtain the insight that is needed to guide the alterations that are possible in changing the relative attractiveness of different occupations? How should such changes be costed or otherwise evaluated for potential managerial use? How will these changes affect career prospects and how might the problems of career management be considered simultaneously with recruitment and related aspects of manpower and personnel planning?
>
> "So much for these indications of potential opportunities in management and the behavioral sciences. In addition to the bearing of these problems on behavioral science and management research and practice, one will also need to address certain kinds of extensions in mathematical modeling. Suppose, for example, that one wishes to insure 'as closely as possible' the possibility of effecting transfer rates subject to constraints on available resources, and that it is desired to obtain a stable desired mix of personnel. We can then pose the problem of redesigning the system transition matrix through financial rewards (and penalties), retraining and other behavioral incentives to achieve this stable mix. Some changes may be possible, some may not, and some may be made only to a limited degree in the matrix and, naturally, all changes may be costly in terms of money or other resources. How should the change be made, or even modelled?
>
> "To rephra[illegible]s mathematically let the vector of desired proportions be represented by π. Next let the desired matrix of transitions be M. Ideally, one would then want, for a steady state, or equilibrium result
>
> $$\pi = M\,\pi. \qquad (1)$$
>
> Of course, in any actual situation one could consider that one has a current transition matrix M_o, and that the possible changes
>
> $$\Delta M = M - M_o \qquad (2)$$

are limited in various ways, e.g., one might have the constraint system

$$G (\Delta M) \geq H \tag{3}$$

where G is a coefficient matrix and H is a matrix of constants, h_{ij}, say, so that (3) limits the possible changes for (2).

We might now pose the following new goal programming problem

$$\begin{aligned} &\text{minimize } w^T \mid \pi - M \pi \mid \\ &\text{subject to } G (\Delta M) \geq H, \end{aligned} \tag{4}$$

and where the constraints (2) are also considered for the weighted absolute value minimization represented in (4). It should be noted explicitly that the constraints in (4) will include budgetary and other constraints on the admissible levels of expenditure for achieving these changes. Alternatively, one might pose the transitions systems redesign so that the objective is to minimize the cost of effecting such changes in order to achieve a matrix that will come within preassigned limits of producing the wanted steady state.

"Naturally, this type of system redesign model can be extended in various ways. For example one such extension might include some provision for achieving such changes over a period of years or in costing the changes in other than the simple linear fashion presented above—e.g., the costs of large changes may be much greater than proportional to smaller ones and total costs may also vary in the stepped sequences by which the changes are effected over time.

"It might be noted that in considering such paths, the effects and costs of training and retraining may also require elaboration. In any event, it may be noted that (4) represents a generalization of the problem of achieving a matrix with a prescribed eigenvector in that the desired matrix is subject to additional conditions. It also generalizes the ordinary problem of steady state analysis in that one can usually only get a goal programming formulation—i.e., an objective involving only getting 'as close as possible' to the desired system—because of the presence of these constraints. Conversely the constraints are designed to include managerial and behavioral considerations and desiderate which can then be studied in terms of their consequences for causing deviations from goals, and or so on.

"In all of the above, a single 'steady state' pattern or approximation is assumed to be desirable for a fixed π. The latter in turn is related to various manpower targets set for the Navy. But this does not end the matter since these targets, in turn, are related to tasks and technologies that may be required or available. Evidently, allowance should then be made for the fact that these changes in personnel mixes can be made (or required) from time to time as when, say, steam-powered ships are replaced by new turbine designs. This suggests, again, a stepped sequence of 'steady states' which may be formalized by replacing the preceding π and M values by π (1), π (2), etc., along with associated matrices M (1), M (2), etc., to conform with requirements in each of a sequence of periods while pursuing an overall optimization in the choice of these matrices."

In case further interest attaches to these kinds of developments we can suggest the reference from which this quotation is taken as providing at least a start. There would be no difficulty, in any event, in extending the model to allow for the reallocation of personnel, just as the new hires are directly allocated to desired posts. However a much more desirable method over the long run might be to study the behavioral implications of current transition rates with the kind of "guided" behavioral-sciences research efforts which we noted at the outset of this reply.

PART III

ASSIGNMENT AND SPECTRAL ANALYSES MODELS

STATIC AND DYNAMIC ASSIGNMENT MODELS WITH MULTIPLE OBJECTIVES, AND SOME REMARKS ON ORGANIZATION DESIGN

by

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Abstract

The assignment model of linear programming is here extended to allow for vector optimizations and dynamic interactions between assigned personnel and positions in ea of which a variety of possible measures and approaches are explored. Formulations involving people-to-peop is well as people-to-position matchings are also examined from the standpoint of organizations in which jobs may be fitted to people or vice versa as well as in weighted combinations. Possible uses of such models for dealing with the problems of placing disadvantaged or handicapped persons are noted, but the analysis stops short of the still further possibilities offered by new types of machine-technology and information systems designs.

This paper served as the basis for a presentation at the Session on "Models of the Firm" in the National Meeting of the Operations Research Society of America in Philadelphia, Pennsylvania, November 7, 1968, and its revision was aided by the discussion at that meeting and by suggestions received from M. M. Flood of the Systems Development Corporation in Santa Monica, and W. Brinckloe of the University Science Center in Pittsburgh, Pennsylvania. Acknowledgement is also due M. G. Simpson and others who participated in the Operational Research Short Course, "Goal and Chance Constrained Programming, New Applications and Developments," sponsored by the Universities of Lancaster and Sussex where this paper was also presented at the Belsfield Hotel in Bowness-on-Windemere, Lancaster, England in June of 1968. Published in *Management Science*, April 1969.

1. Background and Preliminaries

The optimal assignment problem involves (as the name suggests) effecting an assignment of n players to n jobs in such a way as to maximize the total value of the assignment. We can formalize what is involved in the following mathematical formulation:

$$\max z \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij}x_{ij}$$

subject to

$$\begin{aligned} &\sum_{i=1}^{m} x_{ij} = 1 \\ &\sum_{i=1}^{n} x_{ij} = 1 \end{aligned} \qquad (1)$$

when, additionally, the x_{ij} are required to be non-negative integers so that exactly one $x_{ij} = 1$ for each i and j and all other $x_{ij} = 0$. Interpreted verbally, this last requirement means that all persons and jobs are assigned—one person to each job and conversely.

The constants r_{ij} which enter into the figure of merit, z, are supposed to represent "ratings" or "scores" for each man relative to each job.[1] For instance the r_{ij}'s might be provided in the form of relative probabilities for the successful performance of each position by each applicant[2] in order to obtain the "figure of merit," z, which enters into the objective defined by "max, z".[3] Other scoring or rating devices might also be used, of course, and because linear programming optimizations may be extended to general ordered fields,[4] we can use these routes, too, when only rankings or only "qualitative" ratings are wanted. In any event, this interpretation of the assignment problem identifies it with linear programming as a variant of the transportation model,[5] a class of models which have been studied very extensively and for which particularly simple and efficient algorithms are available.

Other interpretations have also been provided which make immediate contacts with still other disciplines and methodologies such as the theory of games,[6] statistical estimation and prediction theory and experimental designs along with related "stochastic" characterizations and extensions.[7] Identification as a

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[1]A variety of devices can be used to reduce other cases to this one. See, e.g., Hillier and Lieberman [36] Chapter 6 or Charnes and Cooper [9] Chapter XIV.

[2]See, e.g., Chapter 1 in M. D. Dunnette [22].

[3]This terminology is adapted from Chapter I in [9].

[4]See either pp. 280 ff. in [9] or pp. 217 ff. in [3].

[5]In fact, any transportation model can also be reduced to an equivalent assignment problem, as observed by Flood and others. See, e.g., Flood [26] and Dantzig [21]. See also Vajda [54] who, following Flood, calls this "the most degenerate transportation model."

[6]Vide, e.g., John von Neumann's original interpretation of this as a zero-sum two-person game of hide and seek, as transcribed by Hartley Rogers and reproduced in [57] pp. 5-12. See also Flood [29] for identification with the traveling salesman problem.

[7]Particular note should be made of the extended work which is reported in King [40].

specially structured version of linear programming—with certain "nice" mathematical properties—rather naturally invited attention to the development of special algorithms which could be used in any application of assignment models.[1] The development of the so-called "Hungarian Method" by H. W. Kuhn[2] provided what was wanted. Simple and efficient for this class of problems, this algorithm would be hard to improve upon to any significant degree. In any event, we do not propose to undertake further investigations of algorithmic possibilities and related mathematical inquiries here. We shall turn rather to other ways of considering personnel assignments in order to examine possible additional directions of development.[3] In particular, we shall attempt to extend the problem specified in (1) to certain vector (as distinguished from scalar) optimizations that will provide contact with aspects of organization-to-individual design[4] and related assignment-to-vocational-guidance considerations.[5] We shall attempt to do this in terms of dynamic as well as static development possibilities in terms of personnel[6] reactions for learning and adjustment on various jobs at different times.[7] To maintain contact with the usual versions of the assignment model, however, we shall not extend these developments here in order to relate assignments to task performances and other environmental factors that might also be treated explicitly in subsequent research.[8]

2. Figures of Merit

As has already been observed, there are many possibilities for choosing the constants r_{ij} as "criterion elements" in (1). Given a choice of r_{ij}'s, however, the resulting scalar "figure of merit," z, is thereby defined and with it an orientation toward an "objective" such as "max. z" seems then to evolve in a rather natural manner.

There is a wider range of choices in the case of vector optimizations. So, we might first examine some of these possibilities as we also simultaneously develop our characterizations of model types and the classes of problems with which they might be associated.

To see what is involved we might first alter our definitions of the r_{ij} and also introduce new constants a_{st} as follows:

$$
\begin{aligned}
r_{ij} &= \text{"amount" of } j^{th} \text{ attribute required of job i.}\\
a_{st} &= \text{"amount" of } t^{th} \text{ attribute possessed by individual s.}
\end{aligned}
\tag{2}
$$

These definitions are motivated by the fact that most jobs involve a variety (i.e., a vector) of specified attributes such as, *e.g.*, previous experience, education and supervisory abilities, and, evidently, persons are usually characterized in terms of a collection of attributes like education, previous salary levels, aptitudes, etc.[9] Of course, some job descriptions may involve attributes that will not be found in any personnel dossier and *vice versa*. In such cases we shall employ the convention that $a_{st} = 0$ or $r_{ij} = 0$ when, say, an individual s possesses an attribute that is not explicitly scored as a requirement for a job, and conversely.[10]

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[1] Cf. Flood [27] for a report on some of the early efforts in this direction and Flood [26] and Munkres [46] for some of the subsequent developments.

[2] See [43] and [44].

[3] Other directions of extension which have previously been essayed include multidimensional (including multicommodity) assignments and nonlinear objective functions. See, e.g., Pierskalla [48] and Whinston-Graves [64].

[4] In the sense of Shelly [18.3]. See also Cyert and MacCrimmon [20] and Shelly and Stedry [49].

[5] E.g., as in McFarland [45] or, for economy-wide assignments, as in Holt and Huber [37] and [38].

[6] In this paper, however, we do not deal with the additional possibilities created by variations in the interactions between persons who might be arrayed in different assignments.

[7] An early example which tries to do this by Markoff-type analyses may be found in [42].

[8] Another path might also be followed, as in [40], to elaborate the usual assignment models in order to deal with statistically varying r_{ij}'s.

[9] See Chapter X in [9] for some descriptions of such "job factors" and "man factors."

[10] Other conventions are also possible.

2.1 Cheb[illegible] or C – Metric

Evidently we cannot expect to be able to fulfill all job requirements. Furthermore, we can choose between "fitting" jobs to individuals as well as "fitting" individuals to jobs.[1] Electing the latter alternative, for purposes of exploring differing objectives, we might proceed via a Chebychev, or C metric,[2] as in the following:

$$\max \mu$$

subject to

$$\begin{aligned} \sum_s x_{is} a_{sj} - r_{ij} - \mu &\geqslant 0 \\ \sum_i x_{is} \qquad\qquad &= 1 \\ \sum_s x_{is} \qquad\qquad &= 1 . \end{aligned} \tag{3.1}$$

Here $x_{is} \geqslant 0$ represents a possible assignment of individual s to job i, and restricting these variables to be integers implies, for the above constraints, that exactly one person will be assigned to each job.

In this formulation the objective is arranged so that the resulting assignments will minimize the maximum deviation between the

$$\sum_s x_{is} s_{sj}$$

and the r_{ij} considered over all i and j, as may be seen by setting $\lambda = -j$ and rewriting the above model in terms of λ,

$$\min \lambda$$

subject to

$$\begin{aligned} \lambda &\geqslant r_{ij} - \sum_s x_{is} a_{sj} \\ 1 &= \sum_i x_{is} \\ 1 &= \sum_s x_{is} \end{aligned} \tag{3.2}$$

$$x_{is} = 0 \text{ or } 1, \quad \text{all } i \text{ and } s,$$

Other developments would carry these analyses further into relations of duality, possible correspondence with constrained games[3] and resulting systems of weights in arranging person-to-job assignments, as well as *vice versa*, and so on. We refrain from undertaking these developments here, however, in order to identify further issues and possibilities.

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[1] Or we can even consider a system in which some jobs are fitted to individuals while in other cases the reverse course is followed and we can also introduce systems of weights, etc., to handle still wider classes of mixture possibilities.

[2] See Appendix A in [9].

[3] Vide, e.g., Chapter XV in [9].

2.2 Weighted Absolute Deviations and ℓ_1 Metrics[1]

Of course, the C metric is not the only possibility. For instance we might employ an ℓ_1 metric and minimize a weighted sum of absolute deviations relative to all assignments. Thus, in the case where we assign a weight of zero to all overfulfillments and a weight of unity to all underfulfillments, the indicated model is

$$\min \sum_i \sum_j \nu_{ij}^-$$

subject to

$$\begin{aligned} &\sum_s x_{is} a_{sj} - \nu_{ij}^+ + \nu_{ij}^- = r_{ij} \\ &\sum_i x_{is} \qquad\qquad\quad = 1 \\ &\sum_s x_{is} \qquad\qquad\quad = 1 \end{aligned} \tag{4}$$

with, of course, $\nu_{ij}^+, \nu_{ij}^- \geqslant 0$ and the x_{is} restricted to integer values of 0 and 1 only.[2]

As a specific example of this formulation we might consider the following system of constraints

$$\begin{array}{rcl} 1 &=& x_{11} + x_{21} \\ 1 &=& \qquad\quad x_{12} + x_{22} \\ 1 &=& \qquad\qquad\qquad\quad x_{13} + x_{23} \\ 1 &=& x_{11} \quad + x_{12} \quad + x_{13} \\ 1 &=& \quad x_{21} \quad + x_{22} \quad + x_{23} \\ \nu_{11}^+ - \nu_{11}^- + r_{11} &=& x_{11}a_{11} \quad + x_{12}a_{21} \quad + x_{13}a_{31} \\ \nu_{12}^+ - \nu_{12}^- + r_{12} &=& x_{11}a_{12} \quad + x_{12}a_{22} \quad + x_{13}a_{32} \\ \nu_{13}^+ - \nu_{13}^- + r_{13} &=& x_{11}a_{13} \quad + x_{12}z_{23} \quad + x_{13}a_{33} \\ \nu_{23}^+ - \nu_{23}^- + r_{23}, && \text{etc.,} \end{array}$$

in order to observe that when a ν_{ij}^+ or a ν_{ij}^- is always part of a basic solution then the x_{is} will have 0 or 1 values as in the usual assignment problem. On the other hand, the maintenance of such bases may raise questions of local optimality and these, too, along with related issues of alternative model formulations and algorithmic developments will need to be dealt with. Such endeavors would turn the present paper away from its main function, however, which is to identify and characterize possible new paths for further research and so we proceed rather as follows to selected other possibilities.

2.3 Non-Archimedean Approaches

The above objectives referred to optimizations which can be interpreted in terms of ordinary distance measures but, especially in personnel and organization design, still other possibilities might be required. Thus.

[1] See Appendix A and Chapter X in [9].
[2] This could, of course, be relaxed with one individual spending part-time on one job and the remainder on one or more of the others.

if we want now to consider such other possibilities we might experiment with non-Archimedean approaches via the nuclei-nucleolus concepts of games as in the following adaptation from Charnes and Kortanek [13]:

$$\min \mu$$

subject to

$$\begin{aligned} &\sum_i \sum_j (x_{is}a_{sj} - r_{ij}) = -\mu_s \\ &\sum U^{\sigma(s)} \mu_s - \mu \geqslant 0, \quad \forall\ \sigma(s) \\ &\sum_i x_{is} = 1 \\ &\sum_s x_{is} = 1. \end{aligned} \tag{5}$$

As before the x_{is} are restricted to be integers and, additionally, the $U^{\sigma(s)}$ are assumed to have non-Archimedean properties–viz., $U^{\sigma(1)} >> U^{\sigma(2)}$ and there exists no number k such that $K\ U^{\sigma(2)} \geqslant U^{\sigma(1)}$.

Even these selected characterizations do not exhaust the possibilities. We may also want to experiment with formulations which maximize "satisfactions" by reference to weighted combinations of each *person's* attributes and the matching potential of each *position's* attributes, along with allocations designed to match person-to-person as well as person-to-job attributes and requirements.[1] We will also need to add constraints that will ensure stipulated minimum levels of fulfillment of job requirements and connect these implicitly to possible changing task requirements.

3. Dynamic Extensions

Instead of pursuing the above topics in more detail we turn to other possible extensions. Most of the research in the area of assignment models has been directed toward static "one-shot" situations. The time-interdependent features of an assignment need also to be considered with explicit attention to ways in which an initial assignment may alter characteristics of individuals as well as jobs. Ways in which such features can be brought into play will now be sketched as follows.

Consider first a simple two-period model in which one explicitly plans to utilize the effect of experiences of individuals on jobs in period one. More precisely, one wants to be able to use this experience for its possible additional advantage in fulfilling job requirements in period 2. Note that the length in periods 1 and 2 need not be the same as when, for instance, period 1 represents a short-run (or break-in) period and period 2 a more permanent longer-run situation. In general one cannot just train to maximum advantage for period 2 but must allow for meeting various requirements in the short run (period 1) well. That is, we want to consider the more general situation in which training, experience, and job preparation and development are all intermingled.

As the simplest such case we first set down a transition matrix for rendering the effect of the i^{th} job in modifying the j^{th} attribute of individual s.[2] Thus, by $a_{sj}^{'}$ we represent the j^{th} attribute of individual s as a result of his job experience in the first period as

$$a_{sj}^{1} = \sum_i x_{is} \left(\sum_k t_{ijk} a_{sk}^{0} \right) \tag{6}$$

---

[1]See the remarks in [11] and [12] as these apply to the U. S. Navy's PADS (Personnel Automated Data System) and other possibilities. The need for such matchings in connection with underprivileged and handicapped persons will be dealt with later in this paper where learning and position alterations are also considered. The pertinence of such considerations may be observed from the discussions in [14] and [47]. See also [31] and [32].

[2]See also the utilization of transition matrices for manpower planning in [11], [12], [42] and [62].

where, as usual, x_{is} is a zero-one variable representing the nonassignment or assignment of individual s to job i. We now assume a^0_{sk} is the amount of attribute k possessed by individual s prior to the first-period job assignment and t_{ijk}, a known constant, represents the effect of job i in altering first-period attribute k to a second-period attribute j. That is, a^1_{sj} represents the effect on attribute j for individual s which is (estimated to result from the assignments that might be made in period 1.

If we now let r^2_{ij} be the requirement of job i for attribute j in period 2, we may assume a set of weights, α_1 and α_2, for use in forming the objective of a new "goal programming" model which we can then represent as follows:

$$\max \alpha_1 \mu_1 + \alpha_2 \mu_2$$

subject to

$$\begin{aligned}
&\sum_s x_{is} a^0_{sj} - \mu_1 - r^1_{ij} \geqslant 0 \\
&\sum_s x_{is} = 1 \\
&\sum_i x_{is} = 1 \\
&\sum_s \sum_k x_{is} t_{ijk} a^0_{sk} - r^2_{ij} - \mu_2 \geqslant 0 \\
&x_{is} = 0,\ 1.
\end{aligned} \tag{7}$$

It might be noted that the term in the double summation in the last constraint may be obtained as follows:

$$\begin{aligned}
\sum_s x_{is} a^1_{sj} &= \sum_s x_{is} \left[\sum_p x_{ps} \sum_k \left(t_{pjk} a^0_{sk}\right)\right] \\
&= \sum_s \sum_k \sum_p x_{is} x_{ps} t_{pjk} a^0_{sk} \\
&= \sum_s \sum_k \sum_p \delta_{ip} x_{is} t_{pjk} a^0_{sk} \\
&= \sum_s \sum_k x_{is} t_{ijk} a^0_{sk}
\end{aligned} \tag{8}$$

where δ_{ip} is the ordinary Kronecker delta, since in this formulation we are assuming every individual is assigned to the same job in both periods.

Next we consider a more flexible formulation in which we are allowing possibilities of changing the job assignments in the second period. To accomplish this we start by observing that variables x^1_{is} and x^2_{is} might be employed. The expression in (8) would then give way to

$$\begin{aligned}
\sum_s x^2_{is} \sum_p x^1_{ps} \sum_k t_{pjk} a^0_{sk} &= \sum_p \sum_s \sum_k x^2_{is}\, x^1_{ps}\, t_{pjk} a^0_{sk} \\
&= \sum_p \sum_s x^2_{is}\, x^1_{ps}\, b^p_{sj}
\end{aligned} \tag{9}$$

where

$$b^p_{sj} = \sum_k t_{pjk} a^0_{sk}.$$

Now we replace the product $x_{is}^2 x_{ps}^1$ by a new zero-one variable y_{ps}^i for which it is sufficient to require

$$0 \leq -y_{ps}^i + x_{ps}^1$$

and (10)

$$1 = \sum_p \sum_s y_{ps}^i$$

in order that it represent the properties of the indicated product.

The new goal programming model may now be written

$$\max \alpha_1 \mu_1 + \alpha_2 \mu_2$$

subject to

$$\begin{aligned}
&\sum_s x_{is}^1 a_{sj}^0 - r_{ij}^{(1)} - \mu_1 && \geq 0 \\
&\sum_s x_{is}^1 && = 1 \\
&\sum_i x_{is}^1 && = 1 \\
&\sum_{s}\sum_{p} y_{ps}^i b_{sj}^p - r_{ij}^{(2)} - \mu_2 && \geq 0 \\
&-y_{ps}^i + x_{ps}^1 && \geq 0 \\
&\sum \sum y_{ps}^i && = 1 \\
&x_{ps}^1, y_{ps}^i, && = 0,\ 1.
\end{aligned}$$

This last model may be extended still further in that the job requirements r_{ij}^2 may be made dependent on the assignments and the requirements in period 1. As an example one might utilize

$$r_{ij}^1 \leq r_{ij}^2 - \gamma_{ij}\left(\sum_s x_{is}^1 a_{sj}^0 - r_{ij}^{(1)}\right)$$

$$r_{ij}^1 \leq r_{ij}^2$$

and still other controls and further effects on these second-period job-attribute requirements are possible.

It should be remarked further that this two-stage model can be easily extended to n stages, at least as a matter of form. There seems to be little point in making such further extensions without the aid of empirical guides. (See, e.g., [19] and [35]. On the further question of relati ·e quality of decisions and hence to task performance see [30].) On the other hand, this kind of guidar. t likely to be forthcoming without opening the kinds of prospects for use and research that have been es ·ove[1] and this, of course, is the main objective of the present paper.

[1] For similar remarks relevant to data developments and modelling for marketing applications see [8].

Summary: Extensions and Conclusions

As already indicated this paper is directed primarily to issues associated with problem identification in order to suggest new possibilities for research and thereby perhaps open further possibilities for applying and extending the management sciences and mathematics. The vigorous research efforts that have been directed to improving the computational power of linear programming, as an example, may have obscured the very important role that mathematical characterizations associated with linear programming have also played[1] in (i) identifying new problems for scientific research and (ii) supplying contacts between apparently disparate parts of management and other activities.[2]

The assignment model is the case in point, but other important examples could be cited also ranging from transportation models through blending models and models of hierarchical (or hierarchoid) type.[3] To be sure, much of this work was initiated from actual applications but this is not the only possibility. Applications may also need to be preceded by research in modelling in order to provide a focus for experience as well as a guide to data requirements and alternative possibilities.[4] This is likely to be the case for an area such as organization design where so little has been accomplished to date–see [18.2]–possibly because explicit and detailed models developed analytically do not appear to be available for this purpose.

Of course, the present paper represents only a start in this direction since it does not include such elements as task requirements, organization environments and other (possibly) relevant features such as the usual hierarchical arrangements, procedural and policy requirements, etc. Nor, does it include problems which relate to the effects of varying amounts and types of information available for appropriate use of optimization techniques in organizational design.[5] The emphasis has been rather on personnel–position interactions in a stipulated array of possibilities as well as on possible ways in which the related vector optimizations might be selected.

The potential of multiple variable techniques such as linear programming might at least be explored as a possible way of developing alternatives to practices which assume only fixed job descriptions along with static organization charts and other arrangements that are prescribed almost independently of the available personnel–with the latter, in turn, being selected on a one-applicant-at-a-time basis. See e.g., [16], [17] and [45]. It seems at least reasonable to suppose that different mixes of personnel might be used to determine how jobs might best be arranged (or rearranged) and described–so that, e.g., "a captain need not know how to navigate his ship provided a sufficient number of his officers or crew can do this adequately." An interesting recent development involves the so-called "buddy system"[6] used by Ford and other automobile companies as a way of dealing with the "inadequate" backgrounds and experiences of some of the "hard-core unemployed." This is at least a step away from the classic principle whereby one supposedly "classifies the job and not the man" and then develops a testing and selection procedure for determining a best fit, if any, from among the persons who might be recruited for these previously arranged and prescribed jobs.

Of course much more that would need to be done can only be hinted at here by reference to the need for models designed explicitly to deal with organization dynamics. This would involve something more than merely allowing for organization-personnel interactions of the kind which we have examined. Something would also be needed in the way of models which resulted in designs that evolve in terms of conditional dynamical systems–with changing job boundaries, changing superior–subordinate relations and possible openings for entirely new jobs (and annihilation of old ones) as persons, tasks or other environmental factors began to alter over time.[7]

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1 See Ijiri [39] for examples and further discussion.
2 For some recent very excellent examples see A. F. Veinott [55] and [56].
3 For further discussion of such model types see Chapter I in [9].
4 *Vide* the discussion of model developments in [8] as a prelude to furthering systematic improvements in marketing data.
5 See discussion in [15].
6 This has, of course, a long history in military training and elsewhere.
7 Note that we have fixed the job and personal ratings in advance but that these could also be related to tasks and other environmental features. Analogies with some parts of the processes of technological change may be observed in [50] and the possible further relations to information alteration as learning proceeds may be observed along the lines of the motivational costing and budgeting procedures described in [51].

An evolution of research and of practice in such directions would involve a great deal besides mathematical modelling, however, and so for the present it has seemed prudent to maintain contact with preceding work on assignment and related models even if this does restrict the present paper, for the most part, to examples which appear to be most germane to the area of personnel administration.

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AN ALGORITHM FOR MULTI-ATTRIBUTE ASSIGNMENT MODELS

AND

SPECTRAL ANALYSES FOR DYNAMIC ORGANIZATION DESIGN

by

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Abstract

A new algorithm for multi-attribute assignment model is presented. Built around the concept of a biased quadratic objective, it proceeds via a sequence of related assignments. Optional stopping is applied when an approximation to the optimum assignment is identified.

A second part of this paper deals with manpower planning and organization designs which interact dynamically. A spectral analysis approach is utilized to obtain estimates of career patterns and aspirations in terms of observed and potential transition problems. Goal programming approaches are utilized in both parts of this paper. This is done not only to relate them to each other but also to relate them these developments to prior modelling efforts which the authors have undertaken in related areas of manpower planning. Implications for further research are sketched in areas such as the volunteer services and related kinds of organizations.

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1. Introduction

The point of departure in this paper is our earlier "Static and Dynamic Assignment Models . . ." coauthored with A. Stedry [8.5]. That paper, ultimately, was directed to problems of dynamic organization design. En route to that objective, it dealt with extensions or ordinary assignment models which included extensions to multi-attribute characterizations of jobs and assignees. In motivation [8.5] was prompted by a recognition that organizations might be designed in response to the multi-attribute characteristics of its personnel rather than, as at present, attempting only to select and train personnel for static and pre-designed organization relations. The resulting organizations might then be further adjusted to changes, and even potential changes, in personnel characteristics. Organizations and even tasks to be performed could be further augmented to accommodate assignments that might be made not only in terms of (1) persons to jobs, but also to (2) decomposition and recombination of job possibilities, as well as (3) past and future contributions to learning from such job combinations along with (4) future experience and training possibilities.

The earlier paper [8.5] was largely directed to identifying and structuring these kinds of possibilities.[1] Its purpose was to open a way for further research and thereby contribute to progress in management science (and its relatives) as well as management practice (and its relatives). Beyond the indicated structurings, no attention was devoted to computational routines, data requirements, etc., of the sort that would be necessary for purposes of managerial implementation.

This will be remedied here, if only in part, by initiating research in algorithmic developments for the assignment portions of the overall models. Note, however, that the usual assignment algorithms will not be adequate for the present case since the jobs and persons are both characterized in terms of sets of attributes. The usual scalar optimizations are then replaced by "goal-programming" varieties in which persons and job characteristics, which are multi-dimensional, are matched "as closely as possible."

As a start toward the wanted algorithmic developments we shall focus on the latter case (i.e., we shall develop our algorithmic suggestions around multi-attribute cases in which an optimization matches job requirements and personnel characteristics "as closely as possible"). Extensions to the dynamics and other algorithmic suggestions, too, are not treated explicitly in this paper.

Many metrics may be selected for gauging the indicated "as close as possible."[2] Here, however, we shall focus on the "ℓ_1 metric" which in turn, corresponds to a use of absolute deviations. Via earlier research[3] the resulting nonlinear problem can be directly stated as a linear programming equivalent. The result is a "goal programming" model which provides access to other models of manpower planning, as in the OCMM series,[4] and also makes present computer codes available for assistance in developing the wanted algorithm.

The above developments will be essayed in the first part of this paper in order, thereby, to sharpen aspects of the already developed "assignment model" extensions to the multi-attribute case. The second part will then be directed to extending some of the previously identified and structured topics in organization design—e.g., in relation to characteristics of available and potential personnel.

Many of the relevant personnel-dynamic organization design consideration are of a probabilistic nature, at best, and hence this, too, needs to be identified and structured in any comprehensive depiction of the

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[1]See [6] for further discussion of modeling techniques for identifying and structuring management problems.
[2]See Appendix A in [4] for further discussion of these metric possibilities.
[3]See [5] and [4].
[4]See [8.0].

problems and prospects for such designs. We do not propose to go into all aspects of these possibilities in the present paper. Instead we shall proceed discursively to identify salient aspects of the indicated possibilities while providing only so much formal structuring as will be needed to clarify intended meanings. To make matters more concrete we shall proceed in terms of a specific organization like, say, an all-volunteer armed service, or force, which one would like to organize in ways that will attract and develop the kinds of persons and relations needed for certain broadly defined "tasks."

The formalizations we shall essay will utilize some of the developments in recent research on mathematical programming approaches to "spectral analysis."[1] Naturally these developments will need to be modified and adapted for the purposes to be served. A natural next step in these developments will then be the creation of "control models" which deal with conformance between plans and realizations.[2] The form which we shall give to these further developments will be directed to relating planned and emergent assignments to longer term considerations of organization design and career management.

2. Algorithmic Developments for Multi-Attribute Assignments

We initiate these algorithmic developments by reproducing the following definitions from the original multi-attribute extension of the assignment model from [8.5]. Therefore, let

$$\begin{aligned} x_{is} &= \text{amount of individual s assigned to job i} \\ r_{ij} &= \text{amount of } j^{th} \text{ attribute required in job i} \\ a_{sj} &= \text{amount of } j^{th} \text{ attribute possessed by individual s.} \end{aligned} \tag{1}$$

The "assignment constraints" are:

$$\begin{aligned} \sum_s x_{is} &= 1 \\ \sum_i x_{is} &= 1 \\ x_{is} &= 0, 1. \end{aligned} \tag{2}$$

The deviations from the goals may be formulated

$$\sum_s x_{is}\, a_{sj} - \nu_{ij}^{+} + \nu_{ij}^{-} = r_{ij}, \tag{3}$$

where the "over" and "under" deviations are given by $\nu_{ij}^{+}, \nu_{ij}^{-} \geq 0$. A relevant objective is to minimize the under fulfillment of these goals via

$$\min \sum_i \sum_j \nu_{ij}^{-}. \tag{4}$$

The above formulation amounts, in essence, to minimizing the deviations from goals in a biased fashion. Of course, the deviations might additionally be weighted in a variety of ways, including, e.g., various uses of preemptive weights.[3]

[1] See [10] and [17].
[2] See [11] and [12]. See also [13].
[3] See the discussion of Non Archimedean approaches in [8.5] as well as [7] and [14].

A practical difficulty with this formulation, as stated, is that it involves a mixed-integer programming problem. The ordinary assignment model, with linear objective function,[1] consists of the system

$$\min \sum_i \sum_s c_{is} x_{is}$$

subject to

$$\sum_i x_{is} = 1 \tag{5}$$

$$\sum_s x_{is} = 1$$

$$x_{is} \geqslant 0.$$

We now observe that this system has the property that any extreme point solution—and *a fortiori* any optimum extreme point—automatically satisfies $x_{is} = 0, 1$. Moreover this property holds even for preemptive or Non Archimedean c_{ij}.

For multi-attribute assignment models of any realistic size this "integer-solution (lattice point) property" can no longer be guaranteed. It therefore becomes desirable to consider new developments such as we shall here undertake in the form of biased-deviation formulations and sequential approximations by means of sequences of ordinary linear programming assignment models. Another desideratum is to arrange matters so that at each sequential stage an approximate solution is at hand with all x_{is} at values of zero or unity.

Although the corresponding formulae can be easily developed for weighted (including preemptively weighted) as well as biased deviations, we shall, for simplicity, here develop only the case of uniform weights in explicit detail.

It is obviously desirable to formulate a goal deviation functional which is convex, and of a simple analytical form. This will result in a global optimum without local optima for the continuous problem in which the mixed integer problem is embedded. This, in turn, will also make it possible to achieve "close-to optimal" zero-one solutions by means of the sequence of approximating linear (possibly preemptive) assignment models.

3. Biased Quadratic Goal-Deviation Functional

We start these developments with what we shall call a "biased, quadratic goal-deviation functional." We shall assume that the r_{ij} and a_{sj} are scaled so that $0 \leqslant r_{ij}, a_{sj} < 1$

$$z_{ij} \equiv \sum_s x_{is} a_{sj}. \tag{6.1}$$

Then a biased quadratic deviation from goal r_{ij} may be represented

$$\left(z_{ij} - r_{ij}\right)^2 - K_{ij}\left(z_{ij} - r_{ij}\right)$$

where (6.2)

$$K_{ij} \geqslant \left(1 - r_{ij}\right).$$

[1] I.e., rather than a goal programming variety (with associated constraints).

Graphically, this amounts to subtracting a linear function

$$y = K_{ij}\left(z_{ij} - r_{ij}\right)$$

from the quadratic deviation function

$$z_{ij} - r_{ij}{}^{2}$$

so as to insure that values of z_{ij} exceeding r_{ij} are more desirable than values $z_{ij} \leqslant r_{ij}$. The choice of K_{ij} should be sufficiently large so that the linear function exceeds the quadratic function for the possible $z_{ij} > r_{ij}$.

Now, subject to the constraints,

$$\begin{aligned} &\sum_s x_{is} = 1 \\ &\sum_i x_{is} = 1 \\ &x_{is} = 0, 1 \end{aligned} \tag{7.1}$$

we wish to minimize

$$Q\,(x, r) \equiv \sum_{i,j}\left[\left(\sum_s x_{is}\, a_{sj} - r_{ij}\right)^2 - K_{ij} \sum_s x_{is}\, a_{sj}\right]. \tag{7.2}$$

Herein we have dropped the constant, $\sum_{ij} K_{ij}\, r_{ij}$, from the linear part since it does not affect the x_{ij} choices under minimization.

For the functional $Q\,(x, r)$, the gradient may be easily computed via

$$\begin{aligned} \frac{\partial Q}{\partial x_{is}} &= \sum_j \left[2\left(\sum_t x_{it}\, a_{tj} - r_{ij}\right) a_{sj} - K_{ij}\, a_{sj}\right] \\ &= \sum_t x_{it}\left(\sum_j 2a_{tj}\, a_{sj}\right) - \sum_j \left(2r_{ij} + K_{ij}\right) a_{sj}. \end{aligned} \tag{8}$$

A sequence of iterations for linear assignment model solutions may now be started. The functional to be minimized at the $(m + 1)^{st}$ iteration is

$$\sum_i \sum_s \frac{\partial Q}{\partial x_{is}^{(n)}}\, x_{is} \tag{9}$$

where $x_{is}^{(n)}$ is the optimal extreme point solution[1] of the n^{th} iteration. Thus, the problem becomes, at this stage

$$\min \sum_i \sum_s \frac{\partial Q}{\partial x_{is}^{(n)}}\, x_{is}$$

with

$$\begin{aligned} &\sum_s x_{is} = 1 \\ &\sum_i x_{is} = 1 \\ &x_{is} \geqslant 0 \end{aligned} \tag{10}$$

[1] Hence $x_{is}^{(n)} = 0, 1$.

where, naturally, the solution gives $x_{is}^{(n)} = 0, 1$ since this is an ordinary assignment problem with a linear functional in the objective.

The procedure can be started with $x_{is}^{(o)} \equiv 0$ and the functional to be minimized as

$$\sum_i \sum_s (-1) \sum_j \left(2\, r_{ij} + K_{ij}\right) a_{sj}\, x_{is}. \qquad (11)$$

Even this first iteration, we observe, may give a good solution $x_{is}^{(1)} = 0, 1$.

From the above it should be clear how this method can be generalized to the case of arbitrarily (or even preemptively) weighted deviations. The result at each iteration will be an assignment model with linear functional. Hence it will have an optimal extreme point with coordinates $x_{is} = 0, 1$.

The above procedure, which is a gradient method operating (at each stage) on a bounded convex linear "flat" can be expected to converge to the face of the convex polytope which contains the optimum solution to the convex problem with the zero-one restrictions. At this juncture it may be expected to start oscillating between the extreme points (with 0, 1 coordinates) which define the edges of the optimal face. These consist of the nearest zero-one points which are the solutions to the continuous convex (quadratic) assignment problem. By recording the value of the quadratic functional after each solution,[1] the progress can be compared and a stop rule devised. At the worst a practical stop rule can be provided in which computations halt when no improvement is obtained in a prescribed number of iterations. Experiments are in order, naturally, prior to the further development of this rule. We anticipate, however, that the zeroth order iteration will, in fact, provide fairly good assignments in many cases.[2]

4. Designs for Career Management and Career Aspirations: A Spectral Analysis Approach

As was noted earlier, such algorithmic developments form only one part of the problems (and opportunities) we are concerned with identifying. Another part involves probabilistic extensions and new directions for studying some of the models that were previously structured (in [8.5]) for considering new kinds of dynamic organization designs. In particular, one may wish to consider the problem of deducing from the observed transition rates and populations—or equivalently the numbers transiting periodically (e.g., yearly)—what "careers" are being pursued and how many persons are pursuing them. For this purpose one may make an analogy between this problem and the spectral problem of resolving a univariate spectrum of energy (or light wavelengths or blood cell counts) into the proportions of elementary distributions (e.g., of Gaussian variety) of which the spectrum is composed. The overall spectrum for us is, however, multi-variate and consists of numbers of people (or transition rates) transiting from position to position in each year over a series of years, together with those occupying these various positions. The elementary distribution consists of "careers," or "truncated careers," which we take to be time sequences of positions occupied by a person (or a corresponding cohort) in an organization.

Within our time horizon we can consider as variables, the numbers of persons starting each career, or a truncated counterpart, at the beginning of each time period. In terms of these variables, knowing the fixed time sequence of jobs in a career, we can develop expressions for the numbers occupying various positions at the beginning of each time period. The number of those occupying each position at the beginning of each time period can also be projected numerically from those initially aboard and the known transition rates. One can then use as the spectral analysis principle the objective of minimizing a weighted sum of the discrepancies between the "on-board" values projected by the variable numbers in the different careers and

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[1]It might be noted that the value of the quadratic functional at the n^{th} stage is given by

$$Q^{(n)} = \sum_{i,j} \left(z_{ij}^{(n)} - K_{ij}\right)\left(z_{ij}^{(n)} - r_{ij}\right)$$

[2]As was observed above.

the numbers which emerge from the known transition rates and the initial job population. Such an analysis provides a new approach to career management based on an objective assessment through data which are independent of the subjective conceptions of careers being pursued by individuals in an ongoing organizational population. This analytical approach can thus provide aid to behavioral sciences research, and management, too, by systematically focusing on discrepancies between career aspirations and organizational realities, including the opportunities admitted by current organization designs.

One potential value which may be secured from this spectral analysis approach is that it can provide an evaluation of careers actually chosen and observed in terms of transitions actually effected. Differences between these objectively observed career patterns and those obtained via, e.g., personal interviews, can then be used to delineate areas for further research into organization design and personal interaction possibilities. Questions that may thus be raised include perception alterations as persons progress over time from the point at which they were introduced into the organization. This, in turn, may provide insight into organization alterations which can then suggest further possible changes in career pattern desires, and so on.

Of course, one must recognize that such expressions of career desires may not be well founded or even capable of complete articulation. This is likely to be an even more compelling consideration in the context of dynamically varying organization designs. In turn, this suggests a reorientation of objectives to allow for evolving contingencies of this variety. A structure different from the Markoff transition varieties may be in order. For example, one might have changes in the demands for transfer from, and to, positions, depending on external or internal factors—including changes in aspirations which reflect changes in perceptions of alternative career patterns. To encompass such ideas the elements of transition matrices may need to be regarded as random variables. This, in turn, would extend the planning process to include planning for transition rates in which numbers of persons and resulting personnel mixes may be known only in probability. Recourse would then be had to models of chance-constrained (total or conditional) variety.

Suppose it is assumed that some control is possible in terms, e.g., of permitted transfers. It should then be possible to erect a conditional-adaptive variety of control model in which an organization can then try to match its task or job requirements by partially controlling (1) the emergent demands for transfers and (2) the new hires for entry into the system. Note that something akin to multi-attribute planning is also needed in these dimensions. The random character of the transition matrices means that these, too, form (partially controllable) elements of a total plan. Furthermore, the predictions will generally vary from the actuality which is subsequently encountered by both parties—*viz*., planners and participants. These discrepancies will evidently then need to be considered in their different aspects for both planners and participants in only partially controllable form for any model which is devised for realistic applications to such situations.

5. Elements for a Spectral Analysis Model

It is not possible to develop all of this at the present moment. Indeed, a great deal of further research will be required to do this. At this juncture it therefore seems best to try to sharpen some aspects of the total problem. This will at least help to delineate what will be required in a more comprehensive research effort. To that end we, therefore, now focus on developing a spectral analysis type of model which might be applied to the observable job transitions and the career pattern provided by an existing organization. To state this differently, we shall focus on the "objective" patterns provided by an existing organization and ignore other aspects of personal aspiration and organization redesign.

In one sense we can consider our spectral analysis model as involved with modeling processes in which each process represents a career. Then we can consider a career as a specified sequence of job positions involving, say, a one-year duration for each position.

To represent this analytically let C_k designate career k via

$$C_k = \left(J_k^1, J_k^2, \ldots, J_k^n\right) \tag{12}$$

where J_k^s is the s^{th} job occupied in the k^{th} career type. We will be considering a fixed period of years. Among the careers to be noted will be some that might be called "truncated careers." Such truncations may begin with a starting job which would only appear later in a non-truncated career.[1] Thus, a truncation of C_k might be

$$C_k = \left(J_k^2, J_k^3, \ldots, J_k^m\right). \tag{13}$$

We shall designate the jobs by numerals $i = 1, \ldots, m$. Thus the J_k^s may be considered as numbers in the range $i = 1, \ldots, m$.

Now suppose $x_k(r)$ represents the number of persons who choose career k in period r. Then, because of these career choices, the number in position i in year $s-1$ is $f_i(s-1)$ where

$$f_i(s-1) = \sum_{k:J_k^{s-1}=i} \sum_{r=1}^{s-1} x_k(r), \tag{14}$$

and the first summation designates the sum over careers k whose position in yearss - 1 is job i.

Let $f_{ij}(s)$ denote the number in position j in year s who were in position i in year s-1 by these choices. Then

$$f_{ij}(s) = \sum_{k:\left(J_k^{s-1} = i;\, J_k^s = j\right)} \sum_{r=1}^{s=1} x_k(r). \tag{15}$$

Let

$$M \equiv \left(M_{ij}\right) \tag{16}$$

denote the organizational transition matrix for the interval of years being considered. Then let $a_i(r)$ denote the number of persons in position i in year r prior to any hiring. Also let

$$b_i(r) = a_i(r) + \text{new hires}. \tag{17}$$

The ratio $f_{ij}(s)/f_i(s-1)$ should approximate M_{ij} or, as a goal, we should like the discrepancy

$$| f_i(s-1) M_{ij} - f_{ij}(s) |$$

to be as small as possible if the career patterns selected by the model are to have at least plausible normative validity. (The vertical strokes indicate an absolute value.)

Using the above considerations, we devise an objective which minimizes the indicated deviations via

$$\min \sum_{i,j,s} | f_i(s-1) M_{ij} - f_{ij}(s) | \tag{18}$$

this minimization is subject to the following constraints,[2]

$$a_i(s) < f_i(s) < b_i(s) \tag{19}$$

[1]It is also possible to extend the notion of career by considering it as a sequence of jobs with a probability or proportional transition from one job to another in a career sequence. We shall not develop this extension here.

[2]It should be borne in mind that the $f_i(s)$ and $f_{ij}(s)$ are known linear functions of the variables $x_k(r)$.

where,[1] it may be recalled,

$$f_i(s) = \sum_{k:J_k^s = i} \sum_{r=1}^{s} x_k(r) \tag{20}$$

and $x_k(r) \geq 0$.

By virtue of previous research,[2] however, this spectral model (which is nonlinear) may be reformulated as a linear programming equivalent. Other normative principles of lesser detail can also be employed, of course, instead of the very precise correspondence implicit in the minimization indicated by (18).

6. Conclusion: Projections for Further Research

In this paper we have regarded the elements M_{ij} of the Markoff matrix as fixed and known. Achieving the "least-deviation" $x_k(r)$ values prescribed by the model then becomes a problem in actual personnel selection and development. Of course, managerial and technological, and other considerations, too, may also enter in an *a priori* fashion via stipulation of the upper and lower bounds which limit the $x_k(r)$ choices in (19). In any event, additional behavioral sciences research will undoubtedly be needed to insure that the persons selected will have the indicated career desires and aspirations.

Such research would constitute a rather natural extension of present practices in personnel practice insofar as personnel selection, career management, etc., are effected to conform "as closely as possible" to the likely career patterns (and related job satisfactions) admitted by a given organization.

The model suggested in this paper and its predecessors (see, e.g., [8] and [9]), admit of other views which also invite attention. One such view would alter the status of the M_{ij} components in M and proceed to regard them as variables with values to be assigned in ways that conform (most closely) to the career aspirations and talents of the participants in an organization. In this view the $x_k(r)$ can be regarded as given. The variables $M_{ij} \geq 0$, $\Sigma M_{ij} \leq 1$ would presumably then be submitted to organization and other constraints.[4] The latter comprehend task requirements as well as their admissible job decompositions. One could even extend these ideas and associate a series of $M_{ij}(t)$ in order to deal with various tasks that might be arranged for an organization evolving over time.[5] Personnel might then also be related to each other for support or learning experiences that could add to the opportunities afforded by a panorama of changing jobs and career opportunities.

Discrepancies between empirically observed $M_{ij}(t)$ and the subjective career aspirations of organization members will naturally invite attention. Analysis of such discrepancies may yield insights and improved forecasts of quit rates or exits as well as new-entry or voluntary reenlistments. Such analyses may also suggest organization changes which will better tap the interests and energies of organization participants.

All of the above point to further research which can be guided and controlled by the kinds of spectral-analysis models we have portrayed in this paper. This research can also be related to other possibilities, too, which we have identified and structured in other papers in this series. A case in point is the "generalized eigen value" approach which we utilized in [9] to indicate how desired equilibrating $M_{ij}(t)$ may be arranged via recruitment, promotions, training, transfers, etc. These roads to organization equilibria are, of course, also pertinent fo the present discussion. By advertising and via recruiting for careers in a given organization, $x_k(r)$

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[1]Observe that these constraints are in so-called "interval programming" form. See, e.g., [1] through [3].
[2]See [4] and [5].
[3]The same remark seems applicable to current practices in career management and analysis. *Vide*, e.g., the illustrative uses of a given Markoff matrix for career analysis and management described by Vroom and MacCrimmon in [16].
[4]Indeed, "goal programming" was originally designed for just such "constrained regression" uses. See, e.g., [4] and [5].
[5]Of course the tasks that are forecasted or desired may be ordered or weighted in a variety of ways to insure that desired goals are achieved (as closely as possible) by specified dates, etc. .

selections may be effected to obtain the requisite numbers in each "career aspiration category" to produce more desirable equilibria (or sequences of equilibria) in the related M_{ij} (t). Success in the research that is thereby indicated should then make it possible to deal simultaneously with manpower planning, career management and organization designs that are coordinated and directed toward producing better performance, better careers and greater job satisfactions in the organizations (military or civilian) which can then use these kinds of approaches.

All of the above, and more, may now begin to be effectual not only in response to the opportunities afforded by these new developments in modeling but also in response to the challenges of persons now demanding more rapid changes in organization that are better designed to attract and hold the efforts of volunteer participants.

This concludes the present paper. Data developments and numerical illustrations for some of these ideas are now being undertaken. These will be presented in some of the subsequent reports we are now readying for release in this OCMM series.[1]

[1]Cf., the e.g., the reports noted under [8.0].

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